Math 1A
Quiz 9 - October 28, 2009

Name: KEY

1. Let $f(x) = xe^{-x^2}$. Answer the following questions about the graph of $y = f(x)$. Please mark your answers **clearly**, and show your work.

   (a) What are the x and y intercepts of the graph?

   (b) What are the horizontal and vertical asymptotes of the graph.

   (c) What are the critical points? For each, say if it is a max / min. (Briefly say why) Is there a point of inflection?

   (d) On what intervals is the function increasing and decreasing

   (e) On what intervals is the function concave up and concave down.

   (f) Sketch the graph.

(a) x-intercepts: If $x \cdot e^{-x^2} = 0$ then $x = 0$ or $e^{-x^2} = 0$, but e^{anything} is never 0, so $x = 0$ is the only x-intercept. At $x = 0$, $y = 0$, so $(0,0)$ is the y-intercept.

(b) There are no vertical asymptotes. The horizontal asymptote is $\lim_{x \to \infty} xe^{-x^2} = \lim_{x \to \infty} \frac{x}{e^{x^2}} \overset{L'H}{=} \lim_{x \to \infty} \frac{1}{2xe^{x^2}} = 0$ and similarly as $x \to -\infty$. So $y = 0$ is a horizontal asymptote.

(c) $y' = e^{-x^2} + xe^{-x^2}(-2x)$ (product rule)

$= e^{-x^2} + 2x^2 e^{-x^2} = e^{-x^2}(1 - 2x^2)$

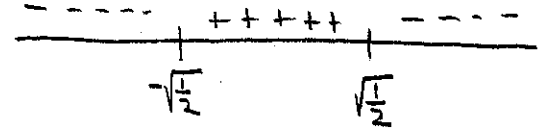
This is zero when $1 - 2x^2 = 0$ or when $x = \pm\sqrt{\frac{1}{2}}$ (critical pts)

max @ $\sqrt{1/2}$, min @ $-\sqrt{1/2}$, use inc/dec

$y'' = -2xe^{-x^2} - 4xe^{-x^2} + 4x^3 e^{-x^2} = e^{-x^2}(-6x + 4x^3) = e^{-x^2} \cdot x \cdot (4x^2 - 6)$

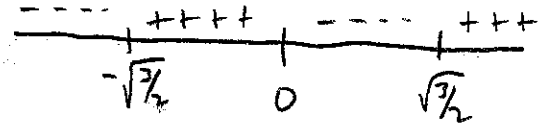
This is zero when $x = 0$ or $x = \pm\sqrt{\frac{3}{2}}$ pts of inflection

(d)



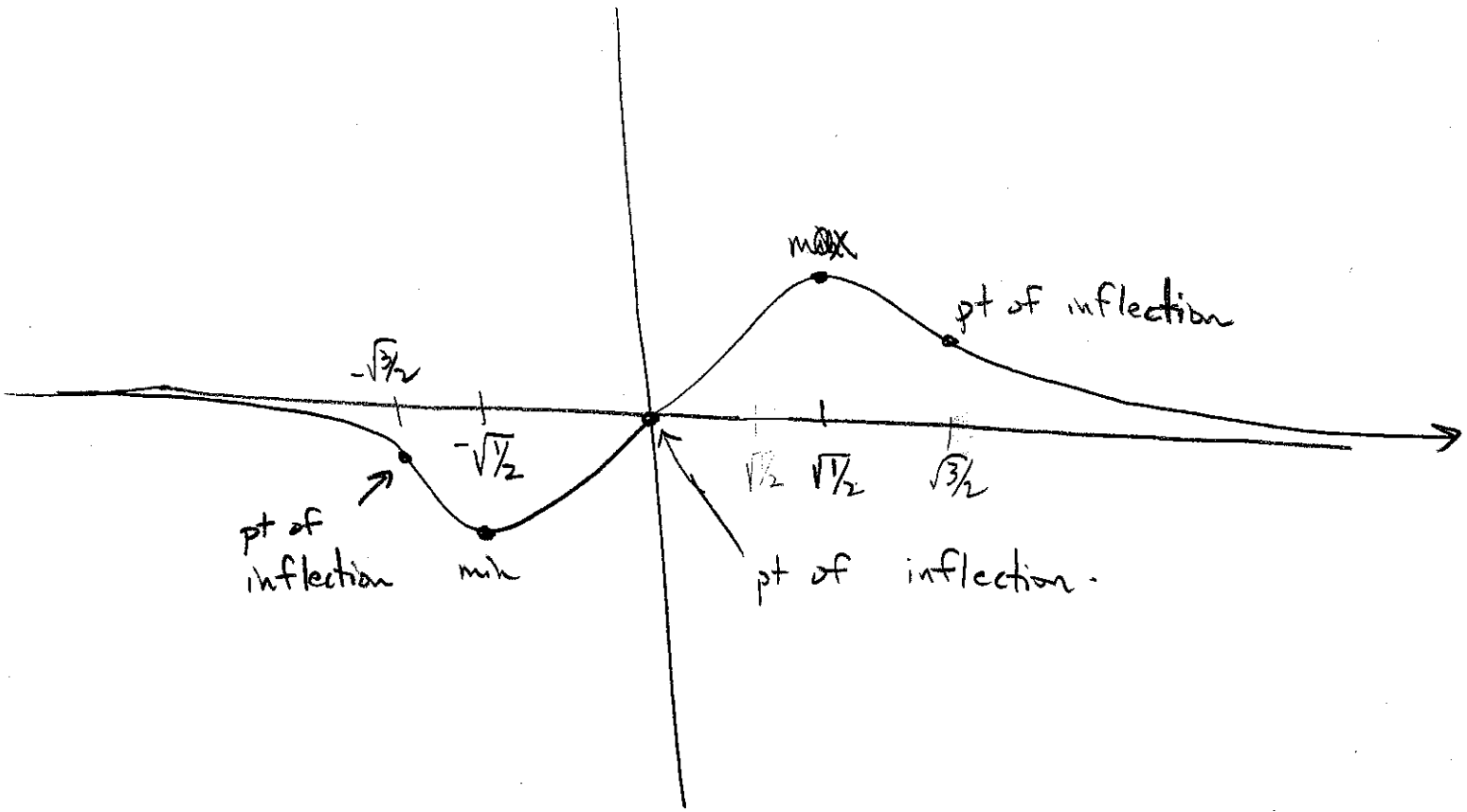
increasing on $(-\sqrt{1/2}, \sqrt{1/2})$ decreasing on $(-\infty, -\sqrt{1/2})$ and $(\sqrt{1/2}, \infty)$

(e)



Concave up on $(-\sqrt{3/2}, 0) \cup (\sqrt{3/2}, \infty)$

concave down on $(-\infty, -\sqrt{3/2}) \cup (0, \sqrt{3/2})$


max
pt of inflection
$-\sqrt{3/2}$
$-\sqrt{1/2}$
$\sqrt{1/2}$
$\sqrt{3/2}$
pt of inflection
min
pt of inflection.