

Math 1A

Quiz 8 - October 21, 2009

Name:

1. A cat decides to climb a tree, but realizes that it must first go to the bank where its account is growing according to the function $y = \sqrt[3]{t}$. Use linear approximation to estimate how much money the cat has after 30 days, ($t = 30$.)

Solution: We want to find a number close to 30 which we can actually compute the cube root of. 27 seems like a good choice. To do linear approximation, we find the tangent line at the point $(27, 3)$. The derivative is $y' = 1/3 \cdot t^{-2/3}$, which when evaluated at $x = 27$ is $1/3 \cdot 1/9 = 1/27$. So the tangent line is

$$y - 3 = \frac{1}{27}(x - 27) \quad y = \frac{1}{27}x + 2.$$

So now we plug in $x = 30$ and get that $y = 30/27 + 2$.

2. After getting the money, the cat decides that climbing trees is lame, and instead decides to state Rolle's Theorem. It writes:

If $f(x)$ is a continuous function on the interval $[a, b]$ and $f(a) = f(b)$ then there is some point c such that $f'(c) = 0$.

Is this correct? Why or why not?

Solution: This is not correct, because Rolle's Theorem requires the function to also be differentiable on the interval (a, b) .

3. The cat then becomes your GSI and decides to be very generous with this problem and grade it on a curve: Here's how it will work. Consider the function:

$$f(x) = x^3 - 3x - 1; \text{ on } [-3, 0]$$

You should give me two numbers a and b . To grade your exam, I will compute $f(a) - f(b)$ and your grade will be proportional to how big this is! In other words, you want to make this difference be as big as possible. Go at it!

Solution: To make this difference the biggest, we want $f(a)$ to be the highest value on the graph, and $f(b)$ to be the lowest value on the graph, so this is just an extreme value problem. We compute the derivative to find critical points:

$$f'(x) = 3x^2 - 3 = 0; \quad x^2 = 1; \quad x = \pm 1.$$

Since we are only looking in the interval $[-3, 0]$ we only consider -1 . We now plug in the endpoints, and the critical numbers:

$$f(-3) = -19, \quad f(-1) = 1, \quad f(0) = -1.$$

So we should pick $a = -1$, $b = -3$. This makes $f(a) - f(b) = 1 - (-19) = 20$.