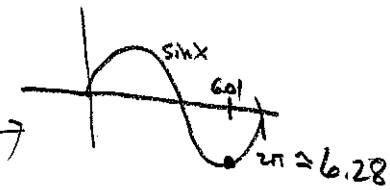
Quiz 4, September 23,2009

Name:

1. Evaluate the following limits. (Be sure to show your work! Remember that $\sin x$ will always be in radians.)

   (a) $\lim\limits_{x\to 6^+} \dfrac{\sin x}{(x-6)(x-7)}$ We plug in 6 and get $\frac{\sin 6}{0}$ so the answer is $\pm\infty$. To see which one, plug in a number just to the right of 6, like 6.01. $\sin 6.01 < 0$ and the bottom terms are $+, -$, so $\dfrac{(-)}{(+)(-)} = +$.



   (b) $\lim\limits_{x\to -\infty} (x + \sqrt{x^2+3x+1})$

$$= \lim_{x\to-\infty} (x+\sqrt{x^2+3x+1})\left(\frac{x-\sqrt{x^2+3x+1}}{x-\sqrt{x^2+3x+1}}\right) = \lim_{x\to-\infty}\frac{x^2-x^2-3x-1}{x-\sqrt{x^2+3x+1}} = \lim_{x\to-\infty}\frac{-3x-1}{x-\sqrt{x^2+3x+1}} = \lim_{x\to-\infty}\frac{-3-\frac{1}{x}}{1-\frac{\sqrt{x^2+3x+1}}{x}}$$

since $x\to-\infty$, $x=-\sqrt{x}$, so

$$= \lim_{x\to-\infty}\frac{-3-\frac{1}{x}}{1+\sqrt{\frac{x^2+3x+1}{x^2}}} = \lim_{x\to-\infty}\frac{-3-\frac{1}{x}}{1+\sqrt{1+\frac{3}{x}+\frac{1}{x^2}}} = -\frac{3}{1+\sqrt{1}} = -\frac{3}{2}.$$

2. If $f(x)$ is a differentiable function, write the "limit" definition of the derivative $f'(x)$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

3. Using any methods you know, find x so that the tangent line to the curve

$$y = 2e^x - 3x + 4\pi^2$$

   is parallel to the line $y = 5x - 17$. $\leftarrow$ slope $= 5$

$y' = 2e^x - 3$

Set $y' = 5$, so

$$2e^x - 3 = 5$$
$$e^x = 4$$
$$x = \ln 4$$

So if $x = \ln 4$, the tangent line will have slope 5 and will be parallel to the given line.