Worksheet 8

1a) $\lim_{t\to 0} \frac{e^{2t}-1}{t} = \frac{0}{0}$

$\stackrel{L'H}{=} \lim_{t\to 0} \frac{2e^{2t}}{1} = \boxed{2}$

b) $\lim_{x\to\infty} \frac{e^x}{x^3} = \frac{\infty}{\infty}$

$\stackrel{L'H}{=} \lim_{x\to\infty} \frac{e^x}{3x^2} \stackrel{L'H}{=} \lim_{x\to\infty} \frac{e^x}{6x} \stackrel{L'H}{=} \lim_{x\to\infty} \frac{e^x}{6} = \frac{\infty}{6} = \boxed{\infty}$

c) $\lim_{x\to 0} \frac{\sin x}{x-1} = \frac{0}{-1} = \boxed{0}$ Don't use L'H

1a) $\lim_{s\to 0} \frac{\ln(1+s)}{se^s} \stackrel{L'H}{=} \lim_{s\to 0} \frac{\frac{1}{1+s}}{se^s + 1\cdot e^s} = \frac{1}{1} = 1$

product rule

b) $\lim_{x\to 0} (1-2x)^{1/x} = L$ (call the limit L)

then $\ln L = \lim_{x\to 0} \frac{1}{x} \ln(1-2x)$

$\ln L = \lim_{x\to 0} \frac{\ln(1-2x)}{x} \stackrel{L'H}{=} \lim_{x\to 0} \frac{\frac{-2}{1-2x}}{1} = -2$

so $\ln L = -2$

$\boxed{L = e^{-2}}$

c) $\lim_{x \to -\infty} x^2 e^x = \lim_{x \to -\infty} \frac{x^2}{e^{-x}} \overset{L'H}{=} \lim_{x \to -\infty} \frac{2x}{-e^{-x}}$

$\overset{L'H}{=} \lim_{x \to -\infty} \frac{2}{+e^{-x}} = \frac{2}{\infty} = \boxed{0.}$

d.) $\lim_{x \to 0} \cos x \sin x = 1 \cdot 0 = 0$

e) $\lim_{x \to \infty} \frac{\ln x}{x^p} \overset{L'H}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{p x^{p-1}} = \lim_{x \to \infty} \frac{1}{p\, x^p} = 0$

2. See attached graphs.

3. long divide:

$$
\begin{array}{r|l}
 & 2x-2 \quad \text{Quotient} \\
2x^2+x-3 & 4x^3-2x^2+5 \\
 & 4x^3+2x^2-6x \\ \hline
 & -4x^2+6x+5 \\
 & -4x^2-2x+6 \\ \hline
 & 8x-1 \quad \text{Remainder}
\end{array}
$$

So $y = \frac{4x^3-2x^2+5}{2x^2+x-3} = \underbrace{2x-2}_{\text{Quotient}} + \frac{8x-1}{2x^2+x-3}$ ← Remainder

So for x really big, $\frac{8x-1}{2x^2+x-3}$ goes to 0 as $x \to \infty$.

$y \approx 2x-2+0$

4. $y = \sqrt{x^2+4x} = \sqrt{x^2+4x+(4-4)}$ complete the square

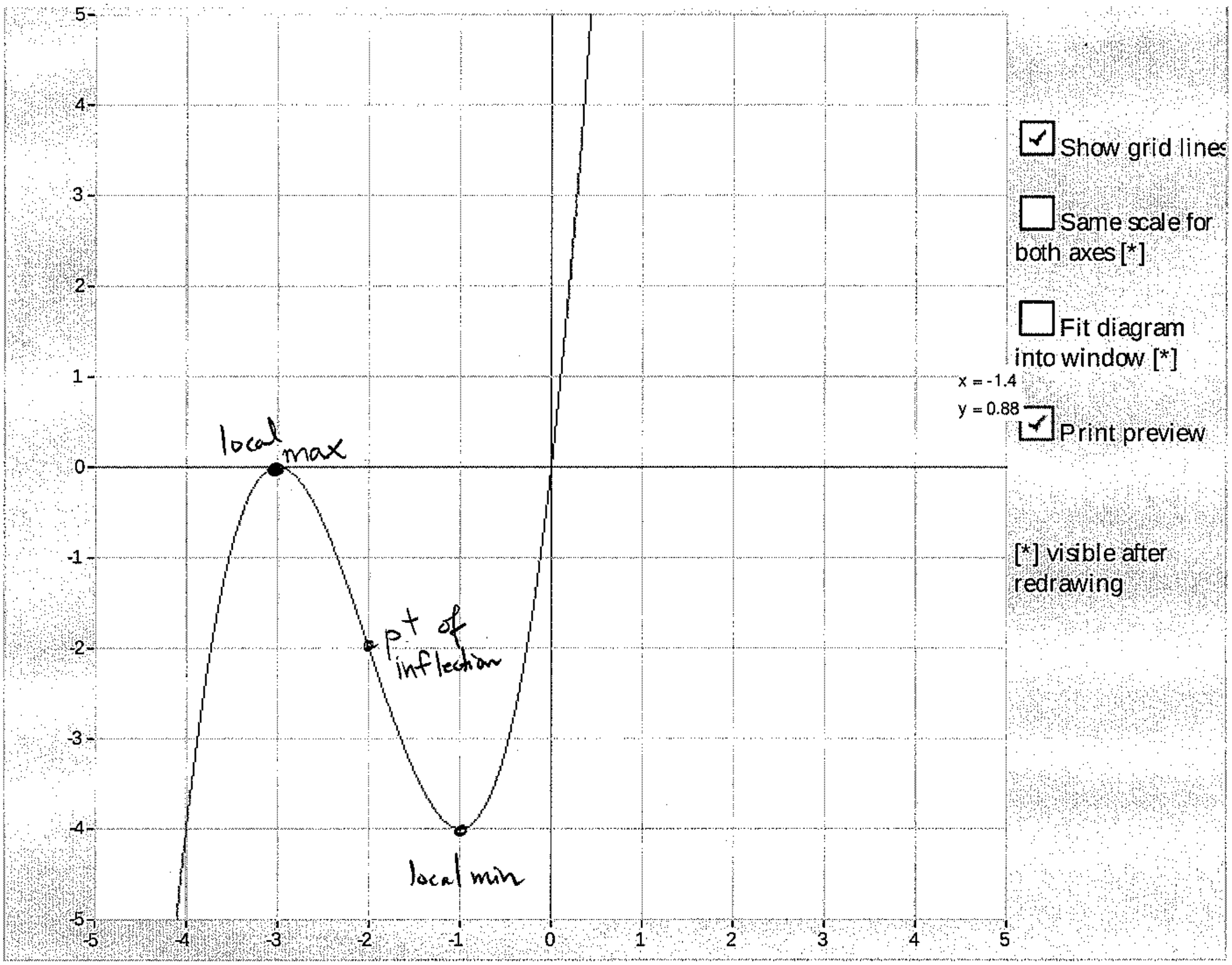
$y = \sqrt{(x+2)^2 - 4}$

for x really big the "-4" part is negligible, so

$y \approx \sqrt{(x+2)^2} = x+2$ if x is positive

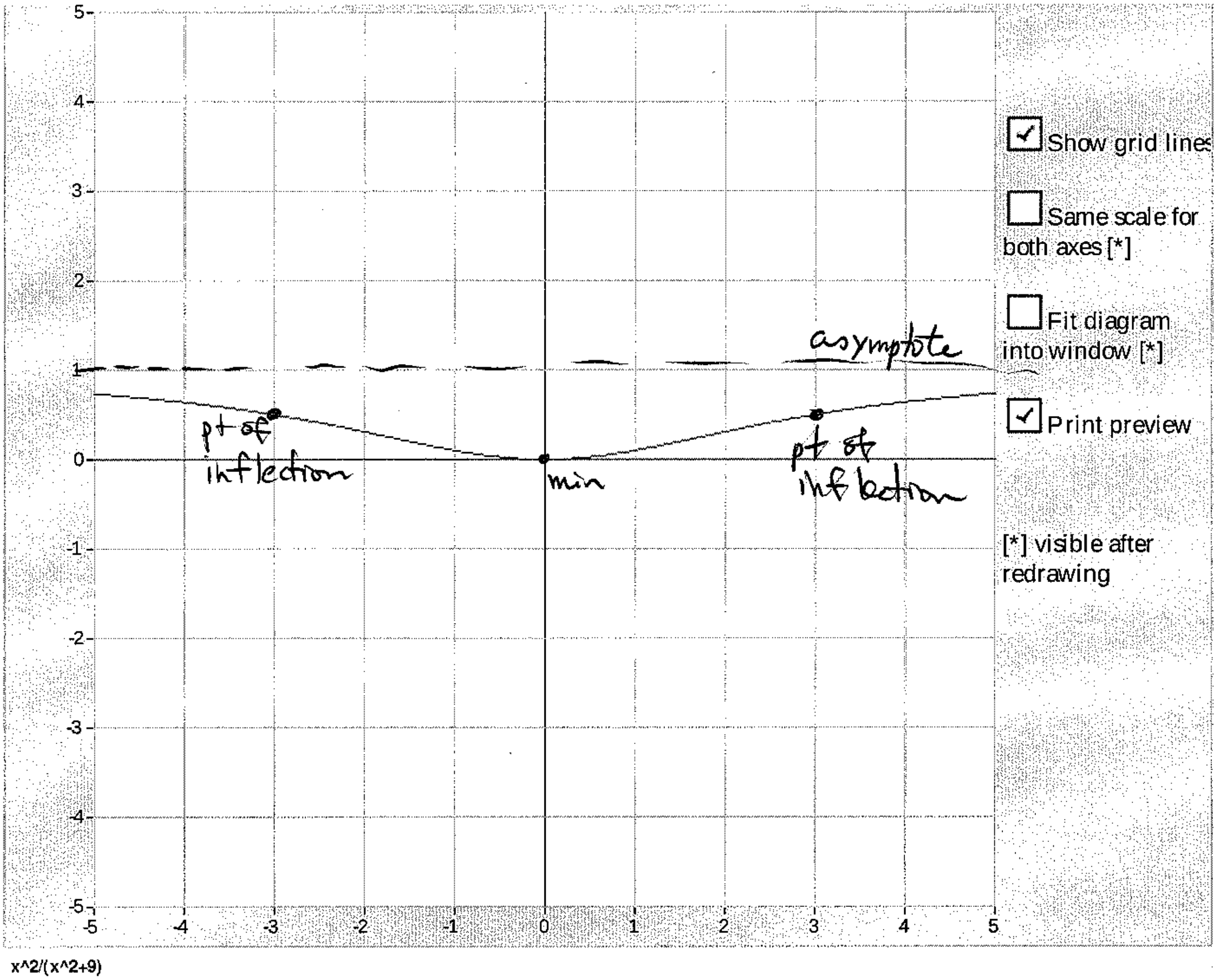
$-(x+2)$ if x is negative

(remember that $\sqrt{x^2}$ is <u>not</u> always $= x$)

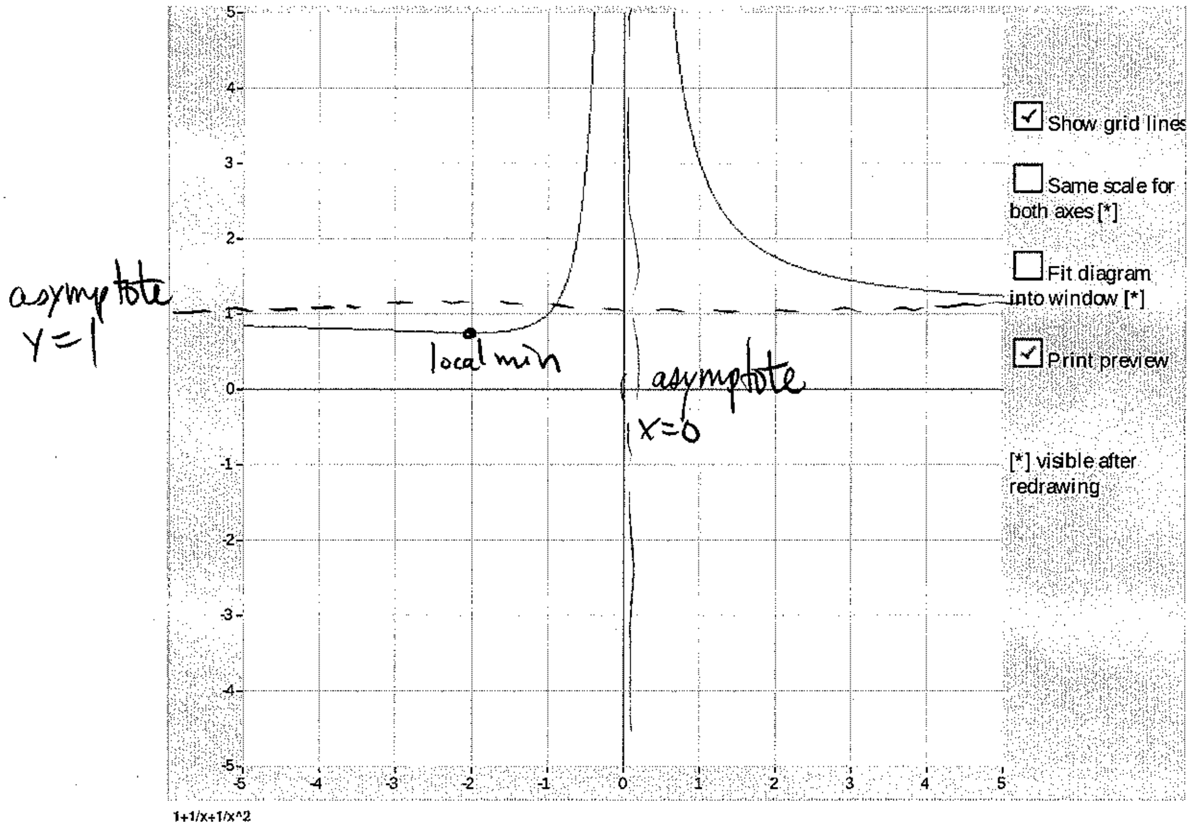


x^3+6*x^2+9*x

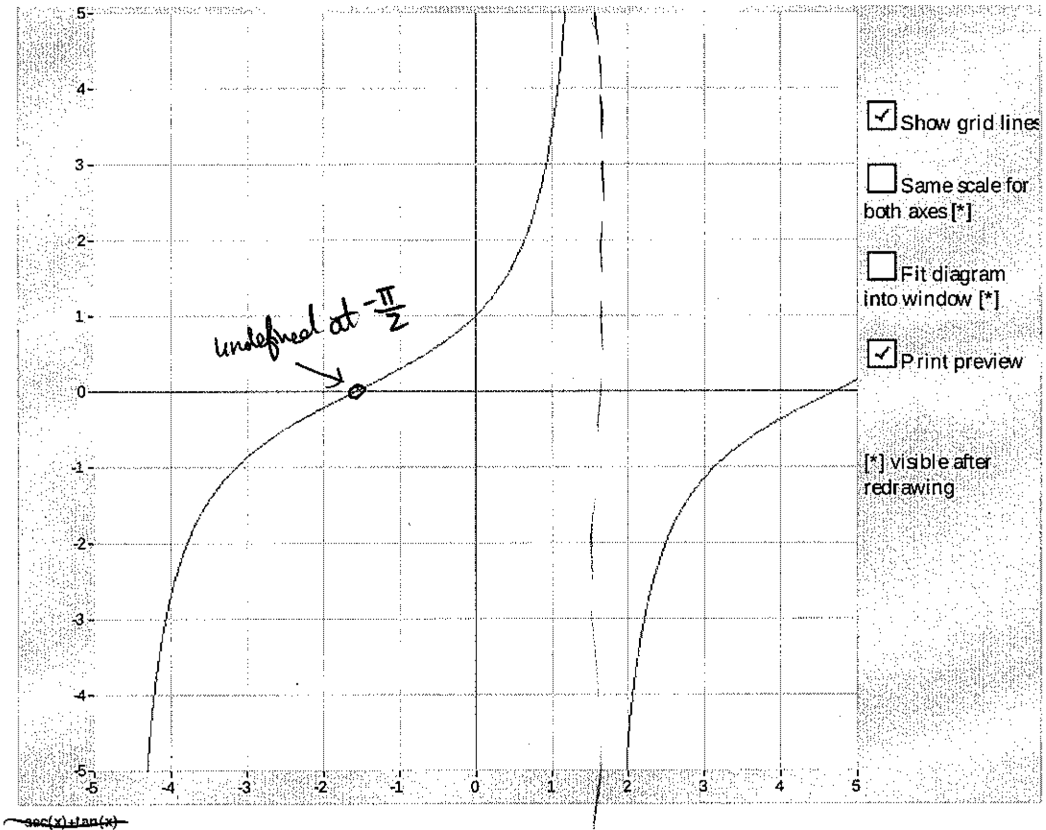
$x^3 + 6x^2 + 9x$


asymptote
pt of inflection
min
pt of inflection
Show grid lines
Same scale for both axes [*]
Fit diagram into window [*]
Print preview
[*] visible after redrawing
x^2/(x^2+9)

$$\frac{x^2}{x^2+9}$$



asymptote
Y=1
local min
asymptote
x=0
Show grid lines
Same scale for both axes [*]
Fit diagram into window [*]
Print preview
[*] visible after redrawing
1+1/x+1/x^2



asymptote $x = \frac{\pi}{2}$

$$\sec x + \tan x = \frac{\sin x + 1}{\cos x}$$

increasing everywhere.

it looks like it has an asymptote at $x = -\frac{\pi}{2}$, but we check that

$$\lim_{x \to -\frac{\pi}{2}} \frac{\sin x + 1}{\cos x} = \frac{0}{0} \overset{L'H}{\Longrightarrow} \lim_{x \to -\frac{\pi}{2}} \frac{\cos x}{-\sin x} = \frac{0}{1} = 0$$