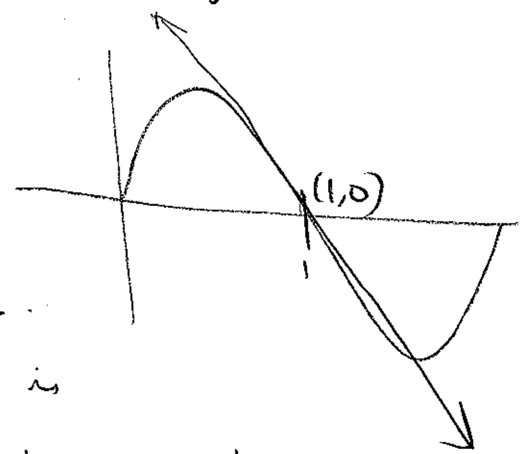
Worksheet 7 Solns

1)

a) top of mt. everest, deepest parts of sea.

b) Tops of hills, bottom of valleys

2) $f(x) = \frac{1}{\pi}\sin(\pi x)$.



$f'(x) = \cos(\pi x)$

So $f'(1) = \cos(\pi) = -1$.

So the eqn of tangent line is

$y = (-1)(x-1)$ or $y = -x+1$

So around the pt $(1,0)$,

$f(x) \approx -x+1$

so $f(0.9) \approx -.9+1 = 0.1$

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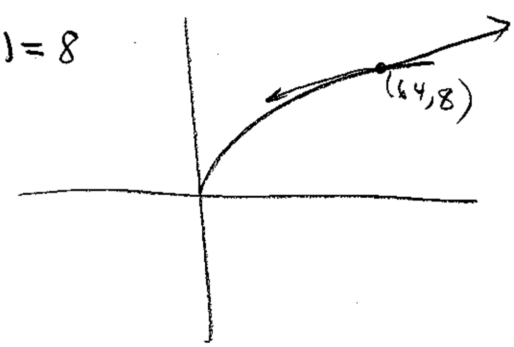
1. $f(x) = \sqrt{x}$ we know $f(64)=8$



$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(64) = \frac{1}{2\cdot 8} = \frac{1}{16}$

So eqn of tangent line is

$y-8 = \frac{1}{16}(x-64)$ or $y = \frac{1}{16}x+4$

So if x is close to 64,

$\sqrt{x} \approx \frac{1}{16}x+4$

So $\sqrt{63} \approx \frac{63}{16}+4 = \frac{127}{16}$.

2. Let $f(x) = \ln(x) + x^2$ We know $f(1) = 0 + 1^2 = 1$

so find tangent line at 1:

$$f'(x) = \frac{1}{x} + 2x$$

$$f'(1) = 1 + 2 = 3$$

so tangent line is

$$y - 1 = 3(x-1) \text{ or } y = 3x - 2$$

so $f(1.1) \approx 3(1.1) - 2 = 3.3 - 2 = 1.3$

3. a) $f(x) = e^{-x} - e^{-2x}$; $[0,1]$

Critical pts: Set $f'(x) = 0$:

$$f'(x) = -e^{-x} + 2e^{-2x} = 0$$

$$-\frac{1}{e^x} + \frac{2}{e^{2x}} = 0$$

$$-e^x + 2 = 0$$

$$e^x = 2$$

$$x = \ln 2.$$ end points

~~So~~ we should check $f(\ln 2)$, $f(0)$, $f(1)$
and see which is biggest/smallest. (use a calculator)

b) $3x^2 - 12x + 5$; $[0,3]$

Crit. pts: $6x - 12 = 0$

$$x = 2$$

So check $f(0) = 5 \leftarrow$ max

$f(2) = -7 \leftarrow$ min

$f(3) = -4$

c) $f(x) = x - \ln x \quad [1/2, 2]$

c.p. $f'(x) = 1 - \frac{1}{x} = 0$

$$x - 1 = 0$$

$$x = 1$$

so check $\left.\begin{array}{l} f(1/2) \\ f(1) \\ f(2) \end{array}\right\} \ldots$

d) $f(x) = \dfrac{x}{x^2+4}$; $[0,3]$

$$f'(x) = \frac{x^2+4-2x(x)}{(x^2+4)^2} = 0$$

so $x^2+4-2x^2 = 0$

$$4 = x^2$$

$x = \pm 2$ in $[0,3]$ only need $+2$

so check

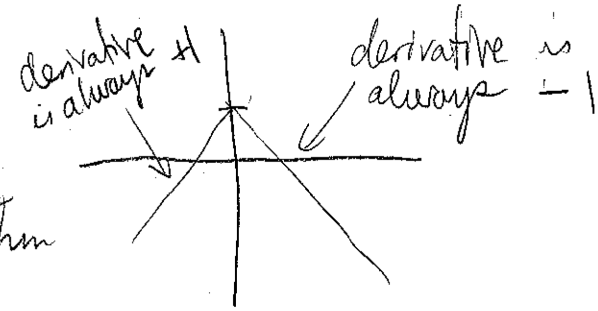
$f(0) = 0 \leftarrow$ min

$f(2) = 1/4$

$f(3) = 3/13 \leftarrow$ max

4.) Remember $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ so

$f(x) = 1 - |x| = \begin{cases} 1 - x & \text{if } x \geq 0 \\ 1 + x & \text{if } x < 0 \end{cases}$



(see picture) This doesn't contradict Rolle's Thm since it is not diff!

5. By the intermediate value thm, and $f(-1) < 0$, $f(1) > 0$, f has at least 1 solution. So now

$f'(x) = 2 + 4 + 6x^2 + 25x^4$

which is always > 0 so the function is always increasing so it only has one zero.

6. $f(x) = 5x^{2/3} + x^{5/3}$

$f'(x) = \frac{10}{3}x^{-1/3} + \frac{5}{3}x^{2/3} = \frac{10}{3}\cdot\frac{1}{x^{1/3}} + \frac{5}{3}x^{2/3}$

this is undefined if $x = 0$.

$f'(x) = 0$ if

$\frac{10}{3}\cdot\frac{1}{x^{1/3}} + \frac{5}{3}\cdot x^{2/3} = 0$

$\frac{10}{3} + \frac{5}{3}\cdot x = 0$

$10 + 5x = 0$

$x = -2$

critical numbers $=$ 0, -2

7. $f(x) = x^3 + x - 1$ is differentiable, so we can apply the MVT, so

$$f(0) = -1$$

$$f(2) = 9$$

So $\dfrac{f(2) - f(0)}{2 - 0} = \dfrac{9 - (-1)}{2} = \dfrac{10}{2} = 5.$

So solve $f'(x) = 5$

So $f'(x) = 3x^2 + 1 = 5$

$$3x^2 = 4$$

$$x = \pm\sqrt{\frac{4}{3}}$$ only want positive root in $[0,2]$

So $\boxed{c = \sqrt{\frac{4}{3}}}$