Worksheet 6 Solutions

1. Set up the exp equation,

$$y = c \cdot e^{kt} \qquad C = \text{starting amount}$$

it doesn't say how much we started with so let's just say it is 1-unit. (Try the problem with another and convince yourself it makes no difference)

So $y = e^{kt}$. How do we find k?

We when $t = 1$ year, we have $.945$ left (94.5% of 1) So

$$.945 = e^{k(1)}, \text{ so } k = \ln(.945)$$

a) Half-life : When will I have $\frac{1}{2}$ remaining?

$$\frac{1}{2} = e^{\ln(.945)t}, \quad t = \frac{\ln(\frac{1}{2})}{\ln(.945)}$$

b) When will it be $.20$?

$$0.20 = e^{\ln(.945)t}, \quad t = \frac{\ln(.20)}{\ln(.945)}$$

2. Derivative!

3. $y = 50e^{kt}$ (50 = amount in 2003)

3-years later, $y = 60$, so $60 = 50e^{k(3)}$

~~so~~ $\frac{60}{50} = e^{3k}$ ~~so~~ $3k = \ln(\frac{6}{5})$, $k = \frac{1}{3}\ln(\frac{6}{5})$

So $y = 50e^{\frac{1}{3}\ln(6/5)t}$. When $t = 32$,

$y = 50e^{\frac{1}{3}\ln(6/5)\cdot 32}$

1. $I = 500 \cdot e^{.20t}$, when $t = 1$,

$I = 500 \cdot e^{20}$

2. In one hour we lost 19,000 (that's 10%) of the population. So in another hour another 10% should die, so 10% of 171,000 is 17,100, so there will be

$$\begin{array}{r} 171000 \\ -\underline{17100} \\ 153900. \end{array}$$

(If my math is right!)

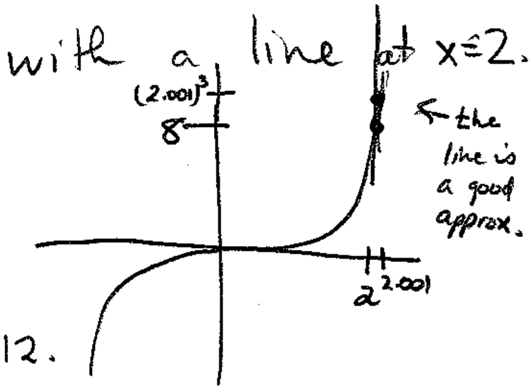
3. $f(x) = x^4 + \ln x$, $a = +1$

$f'(x) = 4x^3 + \frac{1}{x}$. When $a = 1$, $f'(1) = 5$

So the approximation is a line of slope 1 through the point $(1, 1)$ so $y - 1 = 5(x-1)$

$\Rightarrow y = 5x - 4$

4. Let's approximate $y = x^3$, with a line at $x = 2$.



$f'(x) = 3x^2$, $f'(2) = 12$

So through $(2,8)$ we have

$y - 8 = 12(x-2)$ or $y = 12x - 16$

So $(2.001)^3 \approx 12(2.001) - 16 = 24.012 - 16 \approx 8.012$.

5. Since $A = \pi\left(\frac{d}{2}\right)^2 = \frac{1}{4}\pi d^2$ $dA = \frac{1}{2}\pi d \cdot (dd)$,

so "dd" represents the "error" in d for us $dd = .1$

"dA" ——————————— A

So when $d = 25$, $dA = \frac{1}{2}\pi(25)\cdot(0.1)$.

6. $y = \sinh(\tanh x)$

$$y' = \cosh(\tanh x) \cdot \operatorname{sech}^2 x$$

7. $y = \sinh x \cdot \tanh x$

$$y' = \cosh x \cdot \tanh x + \sinh x \cdot \operatorname{sech}^2 x$$

Challenge Problem: Say $f(x) = ax^2 + bx + c$

We want it to pass through $\left(\frac{\pi}{2}, 1\right)$ so $\left(\frac{\pi}{2}\right)^2 a + \left(\frac{\pi}{2}\right)b + c = 1$.

$(\sin(x))' = \cos x$, so at $x = \frac{\pi}{2}$, the deriv. is 0,

So $f'(\pi/2)$ should be 0 too, so $2ax + b\big|_{x=\pi/2} = 0$

So $2a(\pi/2) + b = 0$, and the second derivative of $\sin x$ is

-1 at $x = \pi/2$, so we want $f''(x) = 2a = -1$.

Solve: $a = -1/2$, plug in: $2(-1/2)(\pi/2) + b = 0 \Rightarrow b = \pi/2$

So $\left(\frac{\pi}{2}\right)^2\left(-\frac{1}{2}\right) + \left(\pi/2\right)^2 + c = 1$ so $c = 1 - (\pi/2)^2$.