Limits

Worksheet 4 Solutions

1. $\lim_{x\to -3^-} \frac{x^3-7x}{(x+3)(x+2)}$. When we plug in $x=-3$ we get $\frac{-6}{0}$ so the limit is going to be $\pm\infty$. To figure out which one it is, plug in a number to the left of -3. E.g. -4. Then the signs of top and bottom are

$$\frac{(\text{negative})}{(\text{negative})(\text{negative})} = (\text{neg}) \quad \text{so} \quad \boxed{\lim = -\infty}$$

The limit from the right is $+\infty$ (by the same methods) so the 2-sided limit does not exist.

2. $$\lim_{x\to\infty} \frac{t^2-9}{2t^2+7t+3} = \lim_{x\to\infty} \frac{t^2/t^2 - 9/t^2}{2t^2/t^2 + 7t/t^2 + 3/t^2} = \lim_{x\to\infty} \frac{1-9/t^2}{2+7/t+3/t^2} = \frac{1}{2}.$$

3. $$\lim_{x\to\infty} \frac{x^5+46x^4+9x^2+7}{7x^5+26x^3+8x} = \lim_{x\to\infty} \frac{\frac{x^5}{x^5}+\frac{46x^4}{x^5}+\frac{9x^2}{x^5}+7/x^5}{\frac{7x^5}{x^5}+\frac{26x^3}{x^5}+\frac{8x}{x^5}} = \frac{1}{7}$$

4. $\lim_{x\to\infty} x+\sqrt{x} = +\infty$ since both terms go to $+\infty$

5. $$\lim_{x\to -\infty} x+\sqrt{x^2+x+1} \cdot \frac{x-\sqrt{x^2+x+1}}{x-\sqrt{x^2+x+1}} = \lim \frac{x^2-(x^2+x+1)}{x-\sqrt{x^2+x+1}} = \lim \frac{-x-1}{x-\sqrt{x^2+x+1}}$$

$$= \lim \frac{-\frac{x}{x}-\frac{1}{x}}{\frac{x}{x}-\frac{\sqrt{x^2+x+1}}{x}}$$

now since x is negative (it's $\to -\infty$) we can rewrite $\boxed{x=-\sqrt{x^2}}$, so

$$= \lim \frac{-1-\frac{1}{x}}{1-\frac{\sqrt{x^2+x+1}}{-\sqrt{x^2}}} = \lim \frac{-1-\frac{1}{x}}{1+\sqrt{\frac{x^2+x+1}{x^2}}} = \lim \frac{-1-\frac{1}{x}}{1+\sqrt{1+1/x+1/x^2}} = \frac{-1}{1+\sqrt{1}} = \frac{-1}{2}$$

Derivatives

1. Memorize this for the exam.

2. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $\qquad f(x) = x^3$

$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$

$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h \to 0} 3x^2 + 3xh + h^2$

$= 3x^2$

3. if $f(x) = \sin 3x$ then this limit represents $f'(x)$.

4. $f(x) = e^{3x}$, $a = -2$, so this limit represents $f'(-2)$

5. $y = e^x + 3x^2 - 6$. $\frac{dy}{dx} = e^x + 6x$ and when $x = 0$ this is $e^0 + 6(0) = 1$. So the tangent line has slope 1. Since $f(0) = e^0 + 3(0)^2 - 6 = -5$ the line goes through the pt $(0, -5)$ so the eqn is $y - y_1 = m(x - x_1)$ or $y - (-5) = 1 \cdot x$ or $y = x - 5$.

6.