Math 1A Sections 308-309
Worksheet 3: September 9, 2009

Since we are short on time this week, we won't get through everything on this worksheet, but please try to finish at home what you don't do in class. Thinking about these questions can be mighty confusing, but once you understand what's going on, you'll be able to solve any question that crops up on the exams.

Solutions

1. Write the following down in three different ways, (call the numbers x):

   • Using Intervals• Using inequalities • Using absolute value

   (a) Numbers whose distance from 3 is less than 5.
   $(-2, 8)$, $-2 < x < 8$, $|x - 3| < 5$.

   (b) Numbers whose distance from 3 is less than or equal to 5.
   $[-2, 8]$, $-2 \leq x \leq 8$, $|x - 3| \leq 5$.

   (c) Numbers whose distance from 5 is less than 6.
   $[-1, 11]$, $-1 < x < 11$, $|x - 5| \leq 6$.

   (d) Numbers whose distance from a is less than 1.
   $(a - 1, a + 1)$, $a - 1 < x < a + 1$, $|x - a| < 1$.

   (e) Numbers whose distance from a is less that δ.
   $(a - \delta, a + \delta)$, $a - \delta < x < a + \delta$, $|x - a| < \delta$.

   (f) Numbers that are within δ units of a.
   $(a - \delta, a + \delta)$, $a - \delta < x < a + \delta$, $|x - a| < \delta$. (same as the previous problem)

2. Now write the following sentences using absolute value:

   (a) The distance from x to 3 is less than 5.
   $|x - 3| < 5$

   (b) The distance from $f(x)$ to 4 is less than 1/10.
   $|f(x) - 4| < 1/10$.

   (c) The distance from $f(x)$ to L is less than ϵ.
   $|f(x) - L| < \varepsilon$.

   It is REALLY important that you are able to write these inequalities down correctly, so if you're still stuck, come talk to me.

3. Write down the formal definition of $\lim_{x \to a} f(x) = L$.

   For all $\varepsilon > 0$ there is (or we can find) a $\delta > 0$ such that for $0 < |x - a| < \delta$, $|f(x) - L| < \varepsilon$.

4. Prove that $\lim_{x \to 2} 3x - 4 = 2$.

   I'm not including these proofs because it's not important that your proof matches mine - it's important you understand what your proof is doing and what you are actually proving. This will take time, and something you should talk with me about. But since you're reading this, in your proof, you should choose $\delta \leq \varepsilon/3$

5. Prove that $\lim_{x \to 5} 2x + 5 = 15$.

   Choose $\delta < \varepsilon/2$.

6. (Harder) Can you find those x so that x^2 is within 1 unit of 9?

   (a) Write down the sentence "x^2 is within 1 unit of 9" using absolute value. (Just write it using x^2)
   $|x^2 - 1| < 9$.

   (b) Once you find the answer, write it in interval notation.
   We need $8 < x^2 < 10$, or in other words, $\sqrt{8} < x < \sqrt{10}$.

   (c) What is the center of the interval?
   The center is the midpoint which is $(\sqrt{8} + \sqrt{10})/2$ The center is NOT 3.

   (d) Can you find a smaller interval inside centered around 3?
   Drawing a picture here helps. We know that the distance from 3 to $\sqrt{10}$ is $\sqrt{10} - 3$. Let's call this δ. This is approximately .1622 and the distance from 3 to the left endpoint, is $3 - \sqrt{8}$ which is about .1715. So if we make our interval $3 - \delta < x < 3 + \delta$ then this will be in our old interval.

   (e) Write this interval using absolute value.
   $|x - 3| < \delta$.

   (f) Convince yourself that if x is in this interval, then x^2 is within 1 unit of 9. (This is essentially the whole point of the problem, and you've done all the work by now. The hardest part is just realizing you are done.)