Worksheet 11

① $$\frac{d}{dx}\left(\int_{\cos x}^{x^2} t^2 \ln t \, dt\right) = \left(\underbrace{(x^2)^2 \ln x^2}_{\text{plug in } t=x^2}\right)\cdot\underbrace{(2x)}_{\text{mult by deriv.}} - \left(\underbrace{(\cos x)^2 \ln(\cos x)}_{\text{plug in } t=\cos x}\right)\underbrace{(-\sin x)}_{\text{mult by deriv.}}$$

② $$\int_1^{64} \frac{x^{1/3}+1}{\sqrt{x}}\,dx = \int_1^{64} \frac{x^{1/3}}{x^{1/2}} + \frac{1}{x^{1/2}}\,dx = \int_1^{64} x^{-1/6} + x^{-1/2}\,dx$$

$$= \left(\frac{6}{5}x^{5/6} + 2x^{1/2}\right)\Bigg|_1^{64} = 51.2$$

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① $\bullet$ $$\int x\cos(x^2)\,dx = \int x\cdot\cos(u)\cdot\frac{du}{2x} = \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin u + C$$

$u = x^2$

$\frac{du}{dx} = 2x$ $$= \frac{1}{2}\sin(x^2) + C$$

$dx = \frac{du}{2x}$

② $\bullet$ $$\int \frac{x^{1/3}+2}{x^{2/3}}\,dx = \int \left(x^{-1/3} + 2x^{-2/3}\right)dx = \frac{3}{2}x^{2/3} + 6x^{1/3} + C$$

③ $\bullet$ $$\int \frac{(\ln(x))^2}{x}\,dx = \int \frac{u^2}{x}\cdot x\,du = \int u^2\,du = \frac{1}{3}u^3 + C$$

$u = \ln x$ $$= \frac{1}{3}(\ln x)^3 + C$$

$\frac{du}{dx} = \frac{1}{x}$, $dx = x\cdot du$

• $\displaystyle\int x\sqrt{5x+1}\,dx = \int \frac{u-1}{5}\sqrt{u}\,\frac{du}{5} = \frac{1}{25}\int (u-1)\sqrt{u}\,du$

$u = 5x+1 \qquad x = \frac{u-1}{5}$

$du = 5\,dx$

$dx = \frac{du}{5}$

$$= \frac{1}{25}\int\left(u^{3/2} - u^{1/2}\right)du$$

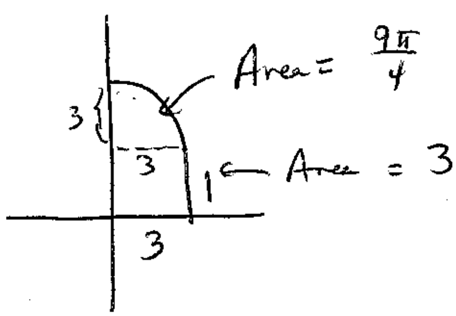
$$= \frac{1}{25}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$= \frac{1}{25}\left(\frac{2}{5}(5x+1)^{5/2} - \frac{2}{3}(5x+1)^{3/2}\right) + C$$

• $\displaystyle\int \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta = \int \frac{1}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}\,d\theta = \int\left(\sec^2\theta + 1\right)d\theta$

$$= \tan\theta + \theta + C$$

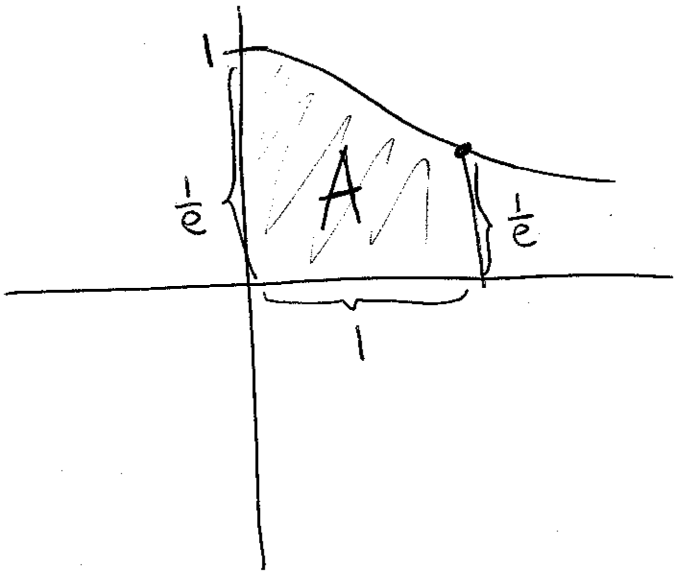
② $\displaystyle\int_0^3\left(1 + \sqrt{9-x^2}\right)dx$



$\therefore \displaystyle\int = 3 + \frac{9\pi}{4}.$

③ $\displaystyle\frac{d}{dx}\left(\int_{\sqrt{x}}^{x^3}\sqrt{t}\sin t\,dt\right) = \left(\sqrt{x^3}\cdot\sin x^3\right)\left(3x^2\right) - \cancel{\sqrt{\sqrt{x}}}\,x^{1/4}\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$

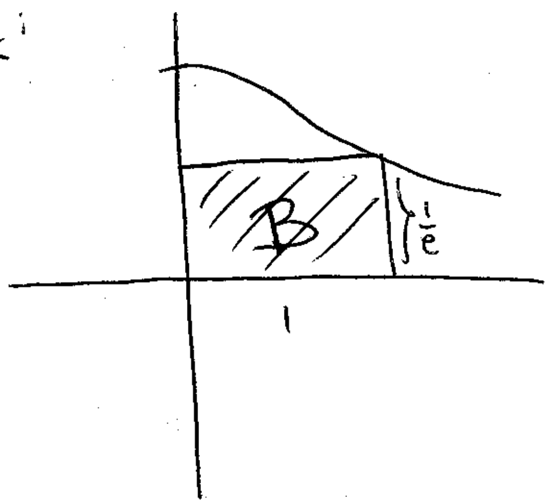
4. Draw a graph:



We want to show

$$\frac{1}{e} \le A \le 1.$$

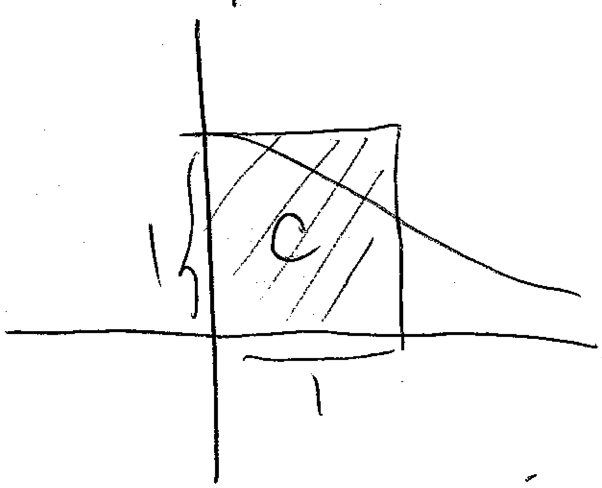
Notice:



$$B \le A$$

$$\frac{1}{e} \cdot 1 \le A$$

And



$$A \le C$$

$$A \le 1 \cdot 1.$$

$$\therefore \frac{1}{e} \le A \le 1.$$