Comments on Worksheet 1

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In general, this worksheet was probably too long, especially with the other things we had on the agenda for today. That being said, I was very happy with how people were working with each other, and I welcome feedback if anyone has any. I think by far the hardest graph on the worksheet was the graph of $\sin(3x-1)$. Many of you had good ideas of how to solve this problem and so I'll present a few of them here in this note.

The main idea in all of these is the following: *The graph will roughly look like a sine wave, only it will probably be shifted, and maybe squished or stretched.* So the goal is **figure out how squished and shifted it is!**

Idea 1: (Brute force = finding zeros)

One group realized that if they knew where the zeros of this function were, then they could guess what the rest of the graph looked like.

How do we find the zeros? Well, sin of something is zero precisely when that something is $0, \pi, 2\pi$, etc. So let's find one of them. If $3x - 1 = 0$ then

$$\sin(3x - 1) = \sin 0 = 0.$$

This happens when $x = 1/3$. So the graph has a zero at $x = 1/3$. Following the same reasoning, if $3x - 1 = \pi$, (or $x = (\pi + 1)/3$) then $\sin(3x - 1) = 0$. So now we know two zeros of the graph, and if we find a few more we can probably "guess" what the whole graph looks like.

Idea 2: (Find the period)

This will be a rough sketch and we'll formalize it in the third solution. One group noticed that normally it takes a period of 2π for the sine curve to repeat, but now since we are taking $3x$ instead of x inside the function, we expect the period to be smaller, probably by a factor of 3, and then there will be a shift... but how do we find it?

This method is actually my favorite when I'm helping people in a trigonometry class, because it's one that always works. Here's how I think about it... Let's call the voice in my head Aldo:

Adam: How do I graph $\sin(3x-1)$? Whoa, that's hard, maybe I can make it simpler...

Aldo: Well how do you usually graph sine functions?

Adam: When I graph $\sin(x)$, I'm lazy and I always just look at the values $0, \pi, 2\pi$. The graph is zero there, and then I just draw in the loops. I call these points the *reference points.*

Aldo: So why don't you just do the same thing here?

Adam: Because I'm taking sine of $3x-1$ and that changes things, doesn't it?

Aldo: Yes, but isn't $\sin 0 = 0$ always? And $\sin \pi = 0$, etc.

Adam: Yea... so?

Aldo: Well why don't you just figure out when the thing inside the parentheses in $\sin(3x-1)$ is zero?

Adam: Well that happens when $3x - 1 = 0$ or $x = 1/3$.

Aldo: So what does that tell you?

Adam: I guess it means that $x = 1/3$ is a zero of my graph and...

Aldo: and that it's one of your reference points!

Adam: Oh ok, so now if I want my second reference point, I should figure out when $3x - 1 = \pi$, and then for the last one, I solve $3x - 1 = 2\pi$.

Aldo: Ok, and then once you have them, you can draw your squiggles.

Idea 3: (Shift and Squish)

When you read the book, you see that there are lots of ways to get new graphs from old ones. For example, on the worksheet today you saw that the graph of the function $\sin(x) - 1$ was obtained from the graph of $\sin x$ by just *shifting the graph down 1 unit.* Similarly, if you had to graph $\sin(x-3)$ you would *shift the graph to the right 3 units.* And if you were asked to graph $\sin 2x$, then the graph will be compressed by a factor of two. But what happens if you have some nasty combination, like in this problem?

It helps to break the problem into multiple steps. *Thought process: I will need to shift and squish, but which one do I do first?* Let's make a new function

$$g(x) = \sin(x - 1).$$

The graph is $g(x)$ is just f shifted one unit to the right.

Now, for mysterious reasons soon to become clear, let's work out what $g(3x)$ is:

$$g(3x) = \sin((3x) - 1) = \text{our function!}$$

So the graph we want, is the graph of $g(3x)$, which is the graph of $g(x)$ *squished* by a factor of 3. And we know how to graph $g(x)$, so we're done.

In reverse:

1. Take the sine graph and move it to the right one unit, (the graph of $g(x)$.)

2. Now squish by a factor of 3, (the graph of $g(3x)$)

This method worked because we knew which combination to try. If we had tried shifting first and then squishing, we wouldn't have ended up with $\sin(3x - 1)$ (What do you end up with?) After lots of practice you can get good with these types of problems, but it's ok to have to try both methods until you get the right answer.

Some of you asked me what sort of details you'd be expected to show on an exam or quiz. I'd be happy with a solution that found the reference points, or even a picture of a squiggle with the words "shift by 1 to the right and then squish by a factor of 3." I realize it's hard to graph these things perfectly and so what's important if that you understand how this curve is different than the standard sine curve.