# An Account of My Qualifying Exam

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May 4, 2009

BS = Bernd Sturmfels, DE = David Eisenbud, DS = Donald Sarason, KOBOK = Katherine O'Brien O'Keeffe

**Commutative Algebra**

- BS: Can you state the Nullstellensatz? (I wrote the general statement down about Jacobsen rings)
- BS: Do you know a ring that isn't Jacobsen ($k[t]_{(t)}$) I finished the statement by saying that if an extension $k < K$ of fields is finite as an algebra then it is finite as a module.
- DE: Prove the second statement follows from the first. (I didn't see how it did. After a few minutes of staring, Bernd said he didn't know how to prove this either and we moved on)
- BS: State the more geometric versions of this (If $k$ algebraically closed, $I \subset k[x_1, \ldots, x_n]$ then $V(I) = \emptyset$ iff $I = (1)$ and $IV(I) = \sqrt{I}$.)
- BS: Prove these follow from the general theorem you've stated. DE: What familiar ring is $R[y]/(fy - 1)$ ($R_f$: I used this in the proof of the NSS)
- BS: Let $I = (x^2 - y^2, x^3 - y^3)$. Find a primary decomposition for I, compute a Grobner basis, compute the Hilbert polynomial by writing a free resolution. Is it Cohen-Macaulay?

**Algebraic Geometry**

- DE: Define projective morphisms and what are they good for? (I said that they were proper, and that pushforwards of coherent sheaves were coherent under projective morphisms)
- DE: What about higher direct images (instantly recanted / not on my syllabus)
- DE: What's your favorite example of a morphism that isn't projective ($\mathbb{A}^1 \to \operatorname{Spec} k$)

- DE: Define Cartier and Weil divisors, relate them to each other. Do you know a Weil divisor which isn't Cartier? (I used the example of the cone over a conic). Compute the Picard group and class group of this space.
- DE: What can we say about curves of degree 4 in $\mathbb{P}^3$? What if they are contained in a plane? What if they are singular? (We then worked through various classifications for genus 0, 1 and 3.)
- DE: This is off your syllabus, but let's try it anyway. Let $X$ be a quartic surface in $\mathbb{P}^3$. Does X contain a curve with negative self-intersection. If you don't know what that means, can the normal bundle have negative degree? (I wrote down the adjunction formula and it turns out this happens only if $X$ is rational.)

**Complex Analysis**

- DS: What is a holomorphic function? What's so nice about them? (I said Cauchy Riemann equations, Cauchy's theorem, that they have a taylor series development)
- DS: How can you prove they have a Taylor series (Talked about removable singularities, Weierstass' theorem, Cauchy's Integral Formula)
- DS: What relationship does Complex analysis have with Algebraic Geometry? (I proved the fundamental theorem of algebra)
- DS: We have only one more question and it comes from the outside member

**Old English**

- KOBOK: Can you translate this passage from Old English into Modern English (it was the story of the Prodigal son and she stopped me after about two lines.)

The exam took just under 2 hours. The committee was extremely nice to me and helped me when I got stuck. I was incredibly nervous before the exam started, but as soon as it started things got much better and I became more relaxed. By the end I was even making jokes! Before the exam started Prof. O'Brien O'Keeffe told me that the secret to success is to relax.