Eigenvalues of a Perturbed Hermitian Matrix

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Abstract

There is a wealth of material on extracting useful information about the eigenvalues of a Hermitian matrix from its submatrices. Chapter 10 of Parlett's ***The Symmetric Eigenvalue Problem*** (in SIAM's Classics in Applied Mathematics series) coherently presented a comprehensive survey of both the classical material named after Cauchy, Courant, Fisher, Lehmann, Weyl, and Wiedlandt, as well as refinements developed in response to demands for the computation of eigenvalues of large matrices by Kahan, Parlett, and others. In this talk, we shall present our small contribution to the effect upon A's eigenvalues if the off-diagonal blocks E and E^* of

$$A = \begin{pmatrix} H_1 & E^* \\ E & H_2 \end{pmatrix}$$

are set to zeros. It is proved that A's eigenvalues λ differ from the eigenvalues $\widetilde{\lambda}$ of

$$\widetilde{A} = \begin{pmatrix} H_1 & 0 \\ 0 & H_2 \end{pmatrix}$$

by no more than

$$\frac{2\|E\|^2}{\eta + \sqrt{\eta^2 + 4\|E\|^2}},$$

where $\|E\|$ is E's spectral norm, and

$$\eta = \begin{cases} \mathsf{dist}(\widetilde{\lambda}, \mathsf{eig}(H_2)), & \text{if } \widetilde{\lambda} \in \mathsf{eig}(H_1), \\ \mathsf{dist}(\widetilde{\lambda}, \mathsf{eig}(H_1)), & \text{if } \widetilde{\lambda} \in \mathsf{eig}(H_2). \end{cases}$$

This improves and unifies two widely known results:

1. $|\lambda - \widetilde{\lambda}| \le \|E\|$, regardless of H_i's spectral distributions;
2. $|\lambda - \widetilde{\lambda}| \le \|E\|^2/\eta$, where $\eta = \mathsf{dist}(\mathsf{eig}(H_1), \mathsf{eig}(H_2))$.