Math 256B. Homework 14

Due Thursday 9 May

1(NC). Let k be a field. Find the dualizing sheaf of $X := V(xy)$ in $\mathbb{P}^2_k$ (the union of two lines in $\mathbb{P}^2_k$ intersecting in a point, with reduced induced subscheme structure).

Either express it as the restriction to X of a line sheaf $\mathscr{L}$ on $\mathbb{P}^2_k$, or show that no such $\mathscr{L}$ exists.

[**Hint:** Don't work too hard.]

2. Let A be a ring. Show that $\operatorname{Tor}_i^A(M, N) \cong \operatorname{Tor}_i^A(N, M)$ for all A-modules M and N, and for all $i \in \mathbb{N}$ (without looking it up anywhere). Use a spectral sequence.

[**Hint:** Use the opposite(s) of one or more categories.]

3(NC). Hartshorne III Ex. 8.1.

4. Hartshorne III Ex. 8.4.

For part (b), you may assume that Remark 7.1.1 is true for arbitrary commutative rings. For part (d), assume that Y is a nonsingular variety.

5(NC). Give an explicit example showing that (III, Thm. 8.8) is false if X and Y are allowed to be locally noetherian schemes instead of noetherian schemes.