

PROBLEM SET 2
MATH 261A

1. Check that F_4 and E_8 satisfy axioms of root systems.
2. Prove that a semisimple Lie algebra \mathfrak{g} is simple if and only if the root system of \mathfrak{g} is indecomposable.
3. Let \mathfrak{g} be a semisimple Lie algebra, \mathfrak{h} be a Cartan subalgebra. For each root α define

$$s_\alpha = \exp(\operatorname{ad}_{X_\alpha}) \exp(-\operatorname{ad}_{Y_\alpha}) \exp(\operatorname{ad}_{X_\alpha}).$$

Prove that $s_\alpha(\mathfrak{h}) = \mathfrak{h}$ and

$$\langle \lambda, s_\alpha(h) \rangle = \langle r_\alpha(\lambda), h \rangle$$

for any $h \in \mathfrak{h}$, $\lambda \in \mathfrak{h}^*$, here r_α denotes the reflection in α^\perp .

4. Prove that among three leg diagrams only E_6, E_7, E_8 are Dynkin diagrams.
5. Prove that among diagrams with one double edge only B_n, C_n and F_4 are Dynkin diagrams.