Conformal and Topological Quantum Field Theories, 276, section 2

August 14, 2007

This is an outline of the course. It is likely that the course will go slower and that only part of this material will be covered. This course will be given in parallel with 276 Section 1, taught by Peter Teichner. Some sections of this syllabus will end up in Section 1 of 276. See this syllabus in late August again for an update.

1 Classical Lagrangian field theories

- Functionals on functions and their variations. Local functionals. Generalization to sections of fiber bundles.
- Classical fields (as sections of fiber bundles) and local Lagrangians.
- Constraints and boundary conditions.
- Space-time categories
- Classical field theory on a space-time category.
- Classical Lagrangian mechanics. Classical mechanics on TM. Constrained mechanical systems.

2 Hamiltonian classical field theory

- The correspondence $(M, \partial M) \to (L(M) \subset M, S(\partial M)$ here $(M, \partial M)$ is an object of a space-time category.
- Gluing
- Correspondence with Lagrangian picture. Examples.

3 Quantization of natural mechanical systems

- Poisson manifolds and their quantization.
- Quantization of symplectic manifolds.
- Physically motivated problems. Scattering, energy levels, spectral densities, bound states.
- The idea of the path integral

4 Quantum field theory

- The correspondence $(M, \partial M) \to (l(M) \subset A(\partial M), A(M))$. Here $A(M)$ quantizes the algebra of function on $S(\partial M)$ and $l(M)$ is a left ideal in $A(M)$ which quantizes $L(M) \subset S(\partial M)$.
- The correspondence $(M, \partial M) \to (z(M) \in H(M), H(partial M)$. It is a result of the previous quantization and a choice of a representation $H(M)$ of the algebra $A(M)$.
- The idea of a path integral. Formal definition.
- Free bose theory. Finite dimensional approximation and the continuum limit.
- Perturbation theory.
- Divergencies in the perturbation theory.
- Regularizations and renormalizations. Finite dimensional approximations and constructive field theory. Finite approximations and scaling limits.
- Semiclassical limit.

5 Classical field theories with fermions

- Grassman algebra and supermanifolds.
- Variational problems involving fermions.
- Lagrangian classical mechanics with fermions.
- Hamiltonian classical mechanics with fermions.
- Classical field theories with fermions

6 Quantum field theories with fermions

- Hamiltonian quantization
- Lagrangian quantization and the perturbation theory.

7 Symmetries in classical and quantum mechanics

- Degenerate functionals and their extrema.
- Poisson action of a Lie group and Poisson reduction. Symplectic reduction.
- Symmetries in quantum mechanics (action of a group by algebra automorphisms).
- Path integral quantization of degenerate Lagrangians. Here one need Fermions. So we will discuss it later.

8 Gauge theories

- Classical theories. Yang-Mills, Chern-Simons.
- Quantization in perturbation theory. Faddeev-Popov and BV quantization.

9 Topological field theories

- Quantization of Chern-Simons via perturbation theory and finite order invariants.
- Axiomatic construction of 3D topological quantum field theories based on surgery and braided monoidal categories.
- The expected relation between two approaches.
- Axiomatic construction of 3D TQFT based on spherical categoreis and triangulations of manifolds.
- The relation between surgery and triangulation construction.

10 Conformal field theories

- Conformal transformations of $\mathbb{R}^n$. Conformal transformations of M_n. Conformal transformations in two-dimensions. Lie algebra and global.
- Classical conformally invariant theories. Liouville and WZW theories.

- The Lie algebra of conformal transformations in two dimensions. Its central extensions.
- Local operators in conformal quantum field theory.
- Construction of CFT from the representation theory.
- Quantum WZW conformal field theory.
- Fusion-braiding data. Conformal field theories and braided monoidal categories.