

University of California, Berkeley
Fall 2021, Math 215A
Midterm 1

Instructor: Prof. David Nadler
GSI: Ethan Dlugie

September 27 to October 1, 2021

Instructions

You may work on this exam at any point between 9:00AM on September 27, 2021 and 11:59PM on October 1, 2021. Before the October 1 deadline, complete and upload your solutions to the three problems below on Gradescope.

You are permitted to use your textbook, your notes, and any other resources that you have produced or that the instructor has provided as part of this course. No external material is allowed.

You are not permitted to discuss these problems with your fellow classmates or anyone else until the solutions have been posted. All work must be your own.

Email the instructor (Professor Nadler) or GSI (Ethan Dlugie) if you have any issues. Otherwise, good luck!

Questions

1. Show that a CW complex is compact if and only if it has finitely many cells.
2. Let X be the quotient space of the cube I^3 by identifying each square face with the opposite square face via a 180° twist. Find a cell structure for X and use it to show that $\pi_1(X) \approx \mathbb{Z}/2\mathbb{Z}$.
3. For $i = 1, 2$, consider the space M_i obtained by identifying two copies of the solid torus $S^1 \times D^2$ along their boundary tori $S^1 \times S^1$ by the map $f_i : S^1 \times S^1 \rightarrow S^1 \times S^1$ where

$$f_1(p, q) = (p, q) \quad \text{and} \quad f_2(p, q) = (q, p).$$

Show that M_1 and M_2 are not homeomorphic.