MATH 215A FALL 2020 MIDTERM 1

Exercise 1. Let X be the union of the unit sphere in $\mathbb{R}^3$ and the segment $\{(0,0,z) : -1 \leq z \leq 1\}$. Let $R : X \to X$ be the self-homeomorphism that sends each point (x,y,z) in X to $(x,y,-z)$. Let T_R be the mapping torus of R; in other words, T_R is the quotient of $X \times I$ by the relation which identifies $(p,0)$ with $(R(p),1)$ for all p in X. Show that the fundamental group of T_R admits a presentation with two generators a,b and one relation $ab = b^{-1}a$.

Exercise 2. Let X be subspace of $\mathbb{C}^2$ consisting of those points (z,w) such that $z^2 \neq w^3$. Let Y be the subspace of $\mathbb{C}^2$ consisting of those points (z,w) such that $z \neq 0$. Show that X and Y are not homeomorphic.

Exercise 3. Let X be a topological space and let $i : A \to X$ be the inclusion of a subspace. Assume that A is path connected, and that the pair (X,A) satisfies the homotopy extension property. Let x_0 be a point in A. Show that there is an isomorphism

$$\pi_1(X/A, [x_0]) = \pi_1(X, x_0)/N$$

where N is the smallest normal subgroup of $\pi_1(X,x_0)$ containing the image of the morphism $i_* : \pi_1(A,x_0) \to \pi_1(X,x_0)$.