1 Problem 1

Suppose that X is a CW complex with exactly one 0-cell $* \in X$. Show that if $X^{n-1} \hookrightarrow X^n$ is nullhomotopic rel $*$, for all $n \geq 0$, then X is contractible.

2 Problem 2

Let $K_1, K_2 \subset \mathbb{R}^3$ be circles such that K_1 is contained in the xy-plane, K_2 is contained in the yz-plane, and K_1, K_2 are linked. For example, you may take K_1 to be defined by the equation $x^2 + y^2 = 1$ and K_2 to be defined by the equation $(y-1)^2 + z^2 = 1$. Let $A = \mathbb{R}^3 \setminus K_1$ and $B = \mathbb{R}^3 \setminus K_2$ and suppose that $\gamma : S^1 \to \mathbb{R}^3$ is a continuous map whose image lands in $A \cap B$. Show that if γ is nullhomotopic in both A and B, then γ is nullhomotopic in $A \cap B$.

3 Problem 3

What are the all the connected 2-to-1 covering spaces of $S^1 \vee S^1$? To receive full credit you must construct all 2-to-1 covering spaces and show that there are no others. (Hint: Every index two subgroup of a group is normal).