Name: ______________ Math 256A Quiz 9 Due Thursday 11/21

1. Show any projective curve C admits a finite map $C \to \mathbb{P}^1$.
2. Consider (closed) points $y_i \in \mathbb{P}^1$, for $i = 1, \ldots, 2k$.
   (a) Construct a smooth projective curve C and a degree 2 map $f : C \to \mathbb{P}^1$ such that f is ramified precisely at the points $f^{-1}(y_i)$, for $i = 1, \ldots, 2k$.
   (b) For fixed points $y_i \in \mathbb{P}^1$, for $i = 1, \ldots, 2k$, is your curve C and/or map f unique? What about for varying points?
3. Let C be a smooth projective curve and $L \to C$ a line bundle. Consider the following condition: (†) for any distinct (closed) points $x \neq y \in C$, there exists a global section σ of L such that $\sigma(x) = 0, \sigma(y) \neq 0$ or vice versa.
   (a) What L on $C = \mathbb{P}^1$ satisfy (†)?
   (b) What L on C of genus 1 satisfy (†)?
   (c) Show for any C, there exists $L \to C$ such that (†) holds.