Welcome to Math 54 Lin Alg & Diff Eq!

<u>Lecture</u> 1: Solving Lin Systems

<u>Enrollment</u> Issues? See Math Dept advisors, 9th floor Evans

<u>This week</u> Friday "Practice Quiz"

<u>Next week</u> Wed Office <u>Hours</u>
12-2pm, 891 Evans

Friday Quiz 1

Why take Lin Alg & Diff Eqs?

It's fun!

It's easy! — $3x - 6 = 0$ (linear!) / $x^2 - x + 3 = 0$ (not linear!)

It's powerful!

3 main goals:

1) Solve $A\underline{x} = \underline{b}$ lin systs.

2) Solve $A\underline{x} = \lambda\underline{x}$ eigenvalue/vector

3) Heat Eqn $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ ... Fourier series

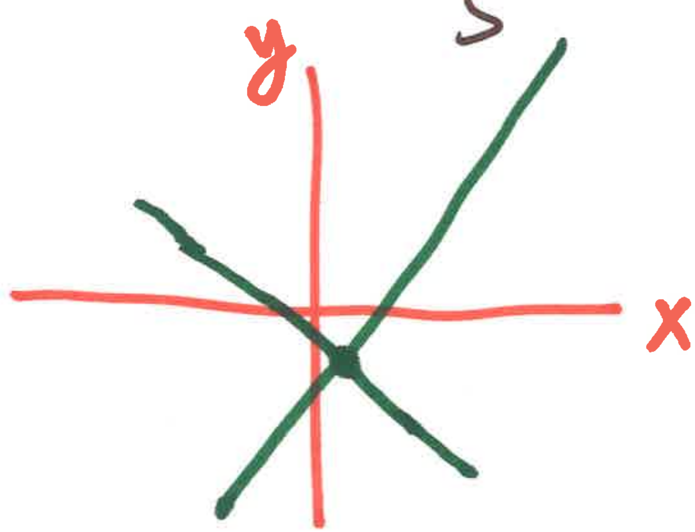
First exer: Show 2) is special case of 1)!

Warmup Solve $\begin{aligned} x - y &= 3 \\ 2x + 3y &= -1 \end{aligned}$

Soln Rewrite syst as

$$\begin{aligned} x - y &= 3 \\ 5y &= -7 \end{aligned}$$

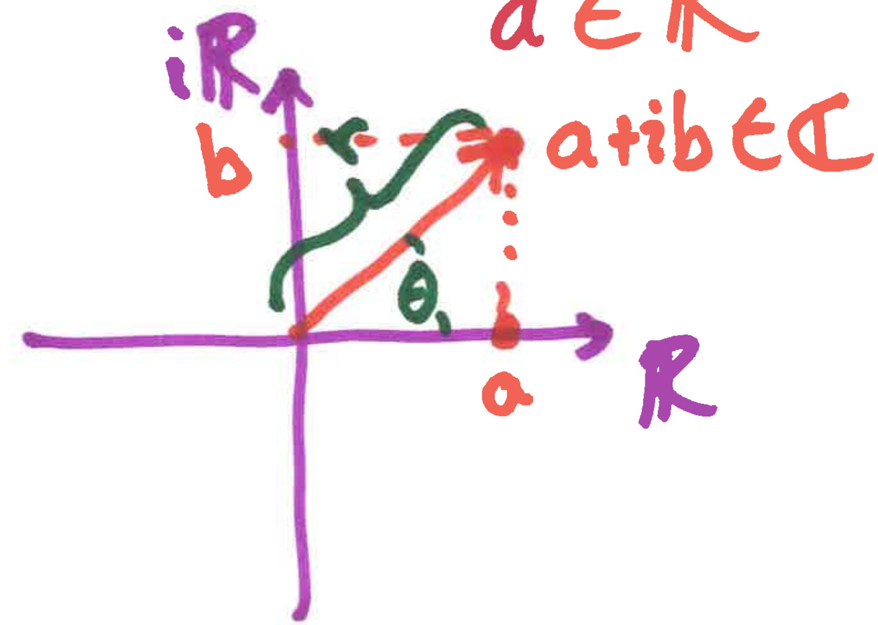
$\boxed{y = -\frac{7}{5}}$ so $x + \frac{7}{5} = 3$ $\boxed{x = \frac{8}{5}}$



Conventions

$\mathbb{R}$ = real numbers

$\mathbb{C}$ = complex numbers



$a = r\cos\theta$
$b = r\sin\theta$

<u>Notation</u>: $a, b, c, \ldots$ numbers; $x, y, z, \ldots$ variables

<u>Def</u> = "Definition", <u>Thm</u> = "Theorem", ...

Def A lin eqn in n variables is any eqn that can be put in form:

$$a_1 \cdot x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

numbers

variables

[redacted]

<u>Exer</u> which are linear?

1) $3(x_1 - 2x_2) = 5(4 - x_3)$ Yes!

2) $3(x_1^2 - 2x_2) = 7$ No!

3) $(x_1 - 3)^2 = x_1^2 + 5x_2$ Yes!

<u>Key idea of lin eqn</u>: Suppose $\underline{b = 0}$.
Then <u>sums</u> and <u>scales</u> of solns are still solns!

__Def__ A __system of lin eqns__ in n variables is a system of eqns that can be put in form

$$\text{m eqns}\left\{\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ \underbrace{a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n}_{n \text{ vars}} = b_m \end{array}\right.$$

Expectations ← careful: only expectations!

$n > m$ many solns

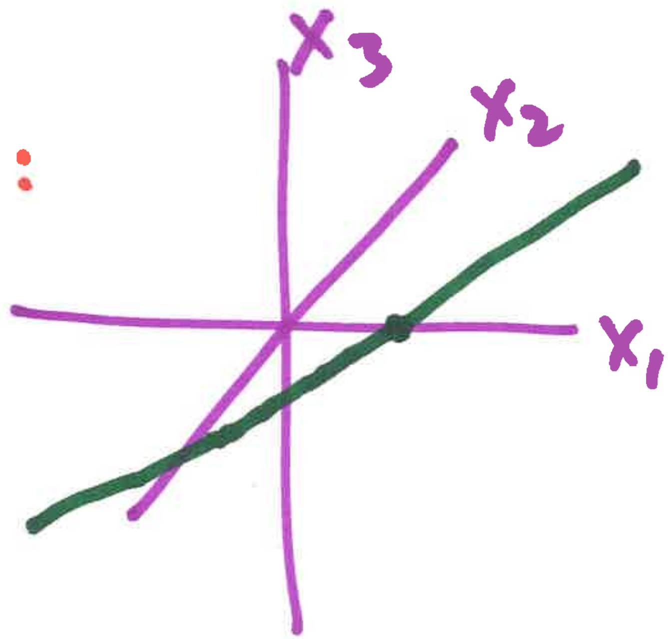
$n < m$ no solns

$n = m$ unique soln

Ex 1) $m=2,\ n=3$

$$x_1 - 3x_2 + 0x_3 = 1$$
$$0x_1 + x_2 - 2x_3 = 0$$

Cartoon:



Set $x_3 = t$ any number

so $x_2 = 2t$

so $x_1 = 1 + 6t$

solns form line!

2) $m=3,\ n=1$

$$-x_1 = 7$$

$$3x_1 = 0$$

$$5x_1 = 2$$

no solns!

Def Soln set of lin syst is set of all n-tuples $(t_1, t_2, \ldots, t_n)$ of numbers that solve system.

Ex 1) Find soln set for

$$3x_1 - x_2 = 0$$
$$2x_1 + 0x_2 = 6$$

Soln set $= \{(3,9)\}$

unique soln!

2) Find soln set for

$$\begin{aligned} 3x_1 + x_2 &= 1 \\ -6x_1 - 2x_2 &= 0 \end{aligned}$$

Soln set = ϕ

no solns!

<u>Challenge</u>: Can you find syst with $m = n = 2$ that has many solns?

Three <u>possibilities</u>:

1) Unique soln

2) No soln

3) Many solns, in fact in this case there will always be ∞-many

<u>Ex</u> For what c does below syst. ~~have~~ fall into each case?

$$x_1 + cx_2 = 1$$
$$2x_1 + 2x_2 = 0$$

t some number

<u>Soln</u>: any soln must be of form $(t, -t)$ by 2^{nd} eqn.

so $t - ct = 1$ by 1^{st} eqn

so $t = \frac{1}{1-c}$

<u>$c \neq 1$</u>: unique soln

<u>$c = 1$</u>: no soln

Def Two lin syst in n vars are equivalent when they have same soln set

Ex For what c are following equiv

I $\quad x_1 - cx_2 = 0$, $\quad x_1 + x_3 = 0$ $\qquad$ solns $= \{(-cs, -s, cs)\}$

II $\quad 2x_1 - x_2 + x_3 = 0$, $\quad x_2 + x_3 = 0$ $\qquad$ solns $= \{(-t, -t, t)\}$

$c = 1$: equiv

$c \neq 1$: not equiv

Matrix notation

$$2x_1 - 3x_2 + 0x_3 - x_4 = 7$$
$$x_1 + x_2 - 3x_3 + 2x_4 = 0$$

$$m\left\{\underbrace{\left[\begin{array}{cccc:c} 2 & -3 & 0 & -1 & 7 \\ 1 & 1 & -3 & 2 & 0 \end{array}\right]}_{n+1}\right.$$

Called: augmented matrix of lin syst

Now: solve lin systs! (when possible ...)

Strategy Change lin syst into **simpler** but **equivalent** lin syst

Then solve!

Using **row ops**

(R1) Add mult of any row to any other

(R2) Interchange rows

(R3) Scale row by nonzero number

Check: (R1-3) don't change soln set