Math 54 Fall 2017 Quiz 4

You have 20 minutes to complete this quiz. To receive full credit, you must justify your answers.

Name : ______________________________

1. (5 points) Find the inverse of the following matrix.

$$\begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}$$

Solution: We proceed by reducing the matrix to RREF after augmenting it with the identity matrix.

$$\left[\begin{array}{ccc|ccc} 7 & 2 & 1 & 1 & 0 & 0 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ -3 & 4 & -2 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 2/7 & 1/7 & 1/7 & 0 & 0 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ -3 & 4 & -2 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 2/7 & 1/7 & 1/7 & 0 & 0 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ 0 & 34/7 & -11/7 & 3/7 & 0 & 1 \end{array}\right] \to$$

$$\left[\begin{array}{ccc|ccc} 7 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1/3 & 0 & 1/3 & 0 \\ 0 & 34 & -11 & 3 & 0 & 7 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 7 & 0 & 5/3 & 1 & -2/3 & 0 \\ 0 & 1 & -1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1/3 & 3 & -34/3 & 7 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 7 & 0 & 0 & -14 & 56 & -35 \\ 0 & 1 & 0 & 3 & -11 & 7 \\ 0 & 0 & 1/3 & 3 & -34/3 & 7 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 8 & -5 \\ 0 & 1 & 0 & 3 & -11 & 7 \\ 0 & 0 & 1 & 9 & -34 & 21 \end{array}\right]$$

Thus, we see that the inverse matrix is

$$\begin{bmatrix} -2 & 8 & -5 \\ 3 & -11 & 7 \\ 9 & -34 & 21 \end{bmatrix}.$$

2. (5 points) Let A be an invertible $n \times n$ matrix. Show that the linear transformation $A : \mathbb{R}^n \to \mathbb{R}^n$ is one-to-one and onto.

Solution: First suppose that $A\mathbf{x} = \mathbf{0}$ for some $\mathbf{x} \in \mathbb{R}^n$. Applying A^{-1}, we get

$$\mathbf{0} = A^{-1}\mathbf{0} = A^{-1}A\mathbf{x} = I_n\mathbf{x} = \mathbf{x}$$

so we see that A is one-to-one.

Now, suppose that $\mathbf{b} \in \mathbb{R}^n$. We want to find a vector in $\mathbb{R}^n$ that is sent to $\mathbf{b}$ by A. Consider the vector $A^{-1}\mathbf{b}$. We have

$$A(A^{-1}\mathbf{b}) = AA^{-1}\mathbf{b} = I_n\mathbf{b} = \mathbf{b}$$

as desired. Thus, we see that A is onto.