Math 54 Fall 2017 Quiz 3

> You have 20 minutes to complete this quiz. To receive full credit, you must justify your answers.

Name : ______________________________

1. (5 points) Find all values of c so that the following matrix has linearly independent columns.

$$\begin{bmatrix} 4 & 7 & c \\ 2 & 4 & 2 \\ 0 & 7 & 3 \end{bmatrix}$$

> **Solution:** The standard method is to reduce the matrix into REF and then to see for which values of c the reduced matrix has a pivot in each column.
>
> In the process of reducing this matrix, it is a good idea to first scale the second row by 1/2 and then switch the first two rows in order to avoid fractions.
>
> $$\begin{bmatrix} 4 & 7 & c \\ 2 & 4 & 2 \\ 0 & 7 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 \\ 4 & 7 & c \\ 0 & 7 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & c-4 \\ 0 & 7 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & c-4 \\ 0 & 0 & 7c-25 \end{bmatrix}.$$
>
> In order to have 3 pivots, we must have $7c - 25 \neq 0$. Thus, the columns are linearly independent when $c \neq 25/7$.

2. (5 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 + x_3 \\ x_1 + x_2 - x_3 \end{bmatrix}.$$

Is T onto?

> **Solution:** T is onto when for any $\mathbf{b} \in \mathbb{R}^2$, $T(\mathbf{x}) = \mathbf{b}$ has a solution. Using the definition of $T(\mathbf{x})$ above, we see that the question is whether the following linear equation system has solution for all $\mathbf{b} \in \mathbb{R}^2$ or not.
>
> $$\begin{bmatrix} x_1 - x_2 + x_3 \\ x_1 + x_2 - x_3 \end{bmatrix} = \mathbf{b}$$
>
> It always has a solution if and only if the coefficient matrix has a pivot position in every row. To check whether this is the case, we row reduce as follows.
>
> $$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \end{bmatrix}$$
>
> We see that there is indeed a pivot in every row. Therefore, T is onto.