You have 20 minutes to complete this quiz. To receive full credit, you must justify your answers.

Name : ______________________________

1. (5 points) Solve the given initial value problem, where $y(\pi) = e^{\pi}$ and $y'(\pi) = 0$.

$$y'' - 2y' + 2y = 0$$

Solution: The auxiliary equation can be constructed as:

$$r^2 - 2r + 2 = 0$$

From which, the roots can be calculated as:

$$r_{1,2} = \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$$

A (complex- valued) solution is

$$y_1(t) = e^{r_1 t} = e^{(1+i)t} = e^t(\cos t + i \sin t) = e^t \cos t + ie^t \sin t$$

So the general solution can be written from the real and imaginary parts as:

$$y(t) = c_1 e^t \cos t + c_2 e^t \sin t.$$

To determine c_1 and c_2, we have to evaluate $y(t)$ and $y'(t)$ at $t = \pi$, so first compute $y'(t)$.

$$y'(t) = c_1(e^t \cos t - e^t \sin t) + c_2(e^t \sin t + e^t \cos t)$$

Then, we evaluate both $y(t)$ and $y'(t)$ at $t = \pi$.

$$y(\pi) = -c_1 e^{\pi} = e^{\pi} \to c_1 = -1$$

$$y'(\pi) = -c_1 e^{\pi} - c_2 e^{\pi} = 0 \to c_1 + c_2 = 0 \to c_2 = -c_1 = 1$$

Thus, the solution to the initial value problem is

$$y(t) = -e^t \cos t + e^t \sin t.$$

2. (5 points) Find a particular solution to the differential equation:

$$4y'' + 11y' - 3y = -2te^{-3t}$$

Solution: The auxiliary equation is

$$4r^2 + 11r - 3 = 0.$$

From which, the roots can be calculated as:

$$r_{1,2} = \frac{-11 \pm \sqrt{121 + 48}}{8} = -3, \frac{1}{4}$$

Since $r = -3$ is a simple root of the auxiliary equation, the particular solution has the form

$$y_p(t) = t(A_1 t + A_0)e^{-3t}$$

To find A_1 and A_0, we have to construct the first and second derivatives and plug them back into the equation, for that:

$$y(t) = (A_1 t^2 + A_0 t)e^{-3t}$$

$$y'(t) = (-3A_1 t^2 + 2A_1 t - 3A_0 t + A_0)e^{-3t}$$

$$y''(t) = (9A_1 t^2 - 12A_1 t + 2A_1 + 9A_0 t - 6A_0)e^{-3t}$$

Plugging these back into the equation will give

$$(-26A_1 t + 8A_1 - 13A_0) = -2t.$$

Thus,

$$-26A_1 = -2$$
$$8A_1 - 13A_0 = 0$$

which implies

$$A_1 = \frac{1}{13}$$
$$A_0 = \frac{8}{169}$$

Finally, the particular solution can be written as

$$y_p(t) = (\frac{1}{13}t + \frac{8}{169})te^{-3t}.$$