Math 54 Fall 2017 Quiz 10

You have 20 minutes to complete this quiz. To receive full credit, you must justify your answers.

Name : ____________________

1. (5 points) Find an orthonormal basis of the subspace H of $\mathbb{R}^4$ below.

$$H = \text{Span}\left\{ \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\4\\4\\1\end{bmatrix}, \begin{bmatrix}4\\-2\\2\\0\end{bmatrix} \right\}$$

Solution: Let v_1, v_2, v_3 be the three vectors given above that span H. We apply the Gram-Schmidt algortihm.

$$u_1 = v_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix} \qquad u_2 = v_2 - \frac{v_2 \cdot u_1}{u_1 \cdot u_1}u_1 = \begin{bmatrix}-3\\2\\2\\-1\end{bmatrix} \qquad u_3 = v_3 - \frac{v_3 \cdot u_1}{u_1 \cdot u_1}u_1 - \frac{v_3 \cdot u_2}{u_2 \cdot u_2}u_2 = \begin{bmatrix}1\\-5/3\\7/3\\-5/3\end{bmatrix}$$

Then $\{u_1, u_2, u_3\}$ is an orthogonal basis for H. To produce an orthonormal basis, we normalize these vectors and obtain the following orthonormal basis of H.

$$\left\{ \frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \frac{1}{3\sqrt{2}}\begin{bmatrix}-3\\2\\2\\-1\end{bmatrix}, \frac{1}{2\sqrt{3}}\begin{bmatrix}1\\-5/3\\7/3\\-5/3\end{bmatrix} \right\}$$

2. (5 points) Find a basis for the orthogonal complement of the following subspace of $\mathbb{R}^4$.

$$W = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 3 \\ 12 \end{bmatrix} \right\}$$

Solution: The orthogonal complement of W is the null space of the following matrix.

$$\begin{bmatrix} 1 & 2 & 5 & 4 \\ 3 & 7 & 3 & 12 \end{bmatrix}$$

Row reducing gives

$$\begin{bmatrix} 1 & 2 & 5 & 4 \\ 3 & 7 & 3 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 5 & 4 \\ 0 & 1 & -12 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 29 & 4 \\ 0 & 1 & -12 & 0 \end{bmatrix}.$$

Thus,

$$\left\{ \begin{bmatrix} -29 \\ 12 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

is a basis of $W^{\perp}$.