Math 54 Fall 2017 Sample Quiz 1

You have 20 minutes to complete this quiz. To receive full credit, you must justify your answers.

Name : ______________________________

1. (5 points) Solve the system of linear equations.

$$2x_1 + x_2 = \frac{1}{2}$$

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Solution: The second equation can be rearranged to get

$$x_1 = \frac{1}{2} - x_2.$$

We plug this into the first equation to get

$$2(\frac{1}{2} - x_2) + x_2 = \frac{1}{2},$$

that is,

$$1 - x_2 = \frac{1}{2}.$$

This gives $x_2 = \frac{1}{2}$.

Then, we plug that in to the second equation to get

$$x_1 + \frac{1}{2} = \frac{1}{2}$$

or $x_1 = 0$. Thus, we have a unique solution

$$x_1 = 0, x_2 = \frac{1}{2}.$$

2. (5 points) Determine the values of C for which the following system of linear equations has one solution, no solutions, and infinitely many solutions.

$$x_1 + Cx_2 = 1$$

$$2x_1 + 6x_2 = 2$$

Solution: Subtract twice the first equation from the second to get

$$(6 - 2C)x_2 = 0.$$

If $C \neq 3$, then $6 - 2C \neq 0$, and we see that $x_2 = 0$. Thus, $x_1 = 1$ and we have a unique solution. If $C = 3$, this gives $0 = 0$ and we see that the second equation is just twice the first. Thus, we can put $x_2 = t$ as a free variable and $x_1 = 1 - 3t$.

To summarize, we have a unique solution $(x_1 = 1, x_2 = 0)$ if $C \neq 3$ and infinitely many solutions if $C = 3$.