Math 54
Fall 2017
Practice Exam 2
Exam date: 10/31/17
Time Limit: 80 Minutes

Name: ______________________________

Student ID: ______________________________

GSI or Section: ______________________________

This exam contains 7 pages (including this cover page) and 7 problems. Problems are printed on both sides of the pages. Enter all requested information on the top of this page.

> This is a closed book exam. No notes or calculators are permitted.
> We will drop your lowest scoring question for you.

You are required to show your work on each problem on this exam. **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

If you need more space, there are blank pages at the end of the exam. Clearly indicate when you have used these extra pages for solving a problem. However, it will be greatly appreciated by the GSIs when problems are answered in the space provided, and your final answer **must** be written in that space. Please do not tear out any pages.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total:	70	

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1. (10 points) Let S be the subspace of $\mathbb{R}^4$ spanned by the following vectors.

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

(a) (2 points) Check that the vectors v_1 and v_2 are orthogonal.

(b) (4 points) Find the orthogonal projection of the vector $w = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$ onto S.

(c) (4 points) Find a basis for the orthogonal complement of S.

2. (10 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) (3 points) Find the eigenvalues of A.

(b) (4 points) Find bases of the eigenspaces of A.

(c) (3 points) Is A diagonalizable?

3. (10 points) Label the following statements as True or False. The correct answer is worth 1 point and a brief justification is worth 1 point. Credit for the justification can only be earned in conjunction with a correct answer. No points will be awarded if it is not clear whether you intended to mark the statement as True or False.

   (a) (2 points) If $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation, $\mathcal{B}$ and $\mathcal{C}$ are bases of $\mathbb{R}^n$, and $[T]_{\mathcal{B}}$ and $[T]_{\mathcal{C}}$ are the $\mathcal{B}$ and $\mathcal{C}$ matrices of T, then $[T]_{\mathcal{B}} = \underset{\mathcal{B}\leftarrow\mathcal{C}}{P}[T]_{\mathcal{C}}$ where $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}$ is the change of coordinates matrix from $\mathcal{C}$ to $\mathcal{B}$.

   (b) (2 points) If B is an echelon form of the matrix A, then the pivot columns of B form a basis of $\mathrm{Col}(A)$.

   (c) (2 points) If A is a 3×3 diagonalizable matrix whose only eigenvalues are 1 and 2, then $(A - I)(A - 2I) = 0$.

   (d) (2 points) The intersection of a subspace W of $\mathbb{R}^n$ and its orthogonal complement $W^\perp$ always has dimension 0.

   (e) (2 points) There is a linear transformation $T : \mathbb{P}_3 \to \mathbb{R}^2$ with $\dim \mathrm{Nul}(T) = 1$ where $\mathbb{P}_3$ is the vector space of polynomials of degree at most 3.

4. (a) (5 points) Suppose W is a subspace of $\mathbb{R}^n$. Show that the transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ given by

$$v \mapsto \mathrm{Proj}_W(v)$$

is a linear transformation, where $\mathrm{Proj}_W(v)$ is the orthogonal projection of v onto W.

(b) (5 points) Show that there is always a basis $\mathcal{B}$ of $\mathbb{R}^n$ with respect to which T is diagonal.

5. (10 points) Find an invertible matrix P and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that the given matrix A has the form $A = PCP^{-1}$.

$$A = \begin{bmatrix} -11 & -4 \\ 20 & 5 \end{bmatrix}$$

6. (10 points) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

   (a) (5 points) Find an orthogonal basis for $\operatorname{Col} A$.

(b) (5 points) Find a 3×3 invertible upper triangular matrix R such that $A = QR$ where Q is a matrix whose columns form an orthogonal basis of $\mathrm{Col}(A)$.

7. (a) (5 points) Suppose that M is an $n \times n$ matrix such that $M^T = M$. If v_1 and v_2 are eigenvectors of M for eigenvalues λ_1 and λ_2, respectively, with $\lambda_1 \neq \lambda_2$, show that v_1 and v_2 are orthogonal.

(b) (5 points) Suppose that A and B are $n \times n$ matrices. Suppose $\mathcal{B} = \{b_1, \ldots, b_n\}$ is a basis of $\mathbb{R}^n$ such that each b_i is an eigenvector of both A and B. Show that $AB = BA$.

Extra space.

Extra space.