Math 256A | Quiz 9 | Fall, 2017

Name: ____________________

1. Inside of $\mathbb{P}^2_{\mathbb{C}}$, consider the scheme E_λ cut out by the homogeneous polynomial

$$f(x,y,z) = y^2 z - x(x-z)(x-\lambda z) \qquad \lambda \neq 0,1 \tag{1}$$

   1. Show E_λ is not isomorphic to $\mathbb{P}^1_{\mathbb{C}}$.

   2. Show $K(E_\lambda)$ is not isomorphic to $K(\mathbb{P}^1_{\mathbb{C}}) \simeq \mathbb{C}(t)$.

   3. Is $K(E_\lambda)$ ever isomorphic to $K(E_\mu)$, for $\lambda \neq \mu$?

2.
   1. Inside of $\mathbb{P}^{n+1}_{\mathbb{C}}$, consider the hypersurface Q_n cut out by a non-degenerate quadratic form. Calculate the rational functions $K(Q_n)$.

   2. Describe the map $Q_n \to \mathbb{P}^{n+1}_{\mathbb{C}}$ as resulting from applying Proj to a homomorphism $S^* \to R^*$ of $\mathbb{Z}^{\geq 0}$-graded $\mathbb{C}$-algebras.

   3. Suppose R^*, S^* are $\mathbb{Z}^{\geq 0}$-graded $\mathbb{C}$-algebras, finitely generated by elements of positive degrees. Does a homomorphism $S^* \to R^*$ of graded algebras always induce a morphism $\mathrm{Proj}(R^*) \to \mathrm{Proj}(S^*)$?