Name: ______________________________

1. 1. Inside of $\mathbb{A}^2_{z\neq 0} = \mathrm{Spec}(k[x,y])$, consider the scheme $E_{z\neq 0} = \mathrm{Spec}(k[x,y]/(g))$ cut out by the polynomial

$$g(x,y) = y^2 - (x-\lambda_1)(x-\lambda_2)(x-\lambda_3) \tag{1}$$

Write a homogenous degree 3 polynomial $f(x,y,z)$ so that setting $z=1$ recovers g.

   2. Inside of $\mathbb{P}^2 = \mathrm{Proj}(k[x,y,z])$, consider the scheme $E = \mathrm{Proj}(k[x,y,z]/(f))$. Calculate the intersection of E with the line at infinity $\mathbb{P}^1_{z=0} = \mathrm{Proj}(k[x,y,z]/(z))$.

   3. Consider the morphism of affine schemes $\phi_{z\neq 0} : E_{z\neq 0} \to \mathbb{A}^1$ given on rings of functions by $k[t] \to k[x,y]/(g)$, $t \mapsto x$. Show $\phi_{z\neq 0}$ extends to a morphism $\phi : E \to \mathbb{P}^1$.

2. Set $\lambda_1 = \lambda_2 = \lambda_3 = 0$. Define a morphism of schemes $\mathbb{P}^1 \to E$ that restricts to an isomorphism $\mathbb{P}^1 \setminus \{[1,0]\} \to E \setminus \{[0,0,1]\}$.

3. Set $\lambda_1 = 1, \lambda_2 = \lambda_3 = 0$. Define a morphism of schemes $\mathbb{P}^1 \to E$ that restricts to an isomorphism $\mathbb{P}^1 \setminus \{[1,0],[0,1]\} \to E \setminus \{[0,0,1]\}$.