Math 256A Quiz 2 Fall, 2017

Name: _____

1. Let $f : X \to Y$ be a map. Briefly explain why each statement is true or give a counterexample.

   a. The pushforward f_* of an injection $0 \to \mathcal{F} \to \mathcal{G}$ of sheaves of abelian groups will again be an injection.

   b. The pushforward f_* of a surjection $\mathcal{F} \to \mathcal{G} \to 0$ of sheaves of abelian groups will again be a surjection.

2. We say a sheaf $\mathcal{F}$ of sets on a space X is locally constant if each $x \in X$ has an open neighborhood $U \subset X$ such that the restriction is a constant sheaf (= sheafification of a constant presheaf).

   What can you say about the associated space $\pi : X(\mathcal{F}) \to X$ of a locally constant sheaf?

3. Consider the open inclusion $j : \mathbb{R}^\times = \mathbb{R}_{<0} \coprod \mathbb{R}_{>0} \to \mathbb{R}$ of the non-zero real numbers.

   Let $\underline{\mathbb{Z}}_{<0} \in \mathrm{Sh}(\mathbb{R}^\times)$ denote the locally constant sheaf of abelian groups with stalk $\mathbb{Z}$ on the negative component $\mathbb{R}_{<0}$ and stalk 0 on the positive component $\mathbb{R}_{>0}$. Similarly, let $\underline{\mathbb{Z}}_{>0} \in \mathrm{Sh}(\mathbb{R}^\times)$ denote the locally constant sheaf of abelian groups with stalk $\mathbb{Z}$ on the positive component $\mathbb{R}_{>0}$ and stalk 0 on the negative component $\mathbb{R}_{<0}$.

   a. Calculate $\mathrm{Hom}_{\mathrm{Sh}(\mathbb{R}^\times)}(\underline{\mathbb{Z}}_{<0}, \underline{\mathbb{Z}}_{>0})$.

   b. Calculate $\mathrm{Hom}_{\mathrm{Sh}(\mathbb{R})}(j_*\underline{\mathbb{Z}}_{<0}, j_*\underline{\mathbb{Z}}_{>0})$.