Math 256A | Quiz 1 | Fall, 2017

Name: ______________________________

1. Let $\mathcal{F}$ be the sheaf of abelian groups on $\mathbb{R}$ that assigns to $U \subset \mathbb{R}$ the abelian group of continuous functions $\sigma : U \to \mathbb{R}$ that vanish on some neighborhood of $0 \in \mathbb{R}$ whenever $0 \in U$.

   Calculate the stalk of $\mathcal{F}$ at $0 \in \mathbb{R}$.

2. Let $\mathcal{F}$ be the presheaf of abelian groups on $\mathbb{R}$ that assigns to $U \subset \mathbb{R}$ the abelian group of continuous, bounded functions $\sigma : U \to \mathbb{R}$.

   Explain why $\mathcal{F}$ is not a sheaf.

3. Let $S^1 = \{z \in \mathbb{C} \mid |z| = 1\}$ be the circle.

   Let $f : \mathbb{R} \to S^1$ be the universal covering $f(x) = \exp(2\pi i x)$.

   Let $\mathcal{F}$ be the sheaf of sets on S^1 that assigns to $U \subset S^1$ the set of continuous sections of f over U. (A section of f over U is a map $\sigma : U \to \mathbb{R}$ such that $f \circ \sigma$ is the identity of U.)

   Set $U_\alpha = S^1 \setminus \{1\}, U_\beta = S^1 \setminus \{-1\}$.

   Calculate the diagram of restriction maps

   $$\mathcal{F}(S^1) \longrightarrow \mathcal{F}(U_\alpha) \times \mathcal{F}(U_\beta) \rightrightarrows \mathcal{F}(U_\alpha \cap U_\beta)$$

   and verify that it exhibits $\mathcal{F}(S^1)$ as an equalizer.