Math 54 Quiz 8

October 16, 2015

1. (a) Let $B = \left\{ b_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$ and $C = \left\{ c_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, c_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$ be bases of $\mathbb{R}^2$.
   Find the change-of-coordinates matrix from B to C.

   (b) Let x=$\begin{bmatrix} 5 \\ 10 \end{bmatrix}$. Using the fact that $[x]_B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, find the coordinates of x with respect to the C basis.

   Solution: (a) P from B to C is $\begin{bmatrix} 7/5 & 4/5 \\ -6/5 & -7/5 \end{bmatrix}$.

   (b) Multiply P on the right by $[x]_B$ to get $[x]_C = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$. A simple row reduction lets you use this answer to check your matrix from part (a). Make sure you see why this is true.

2. Find the eigenvalues of $A = \begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix}$ and one corresponding eigenvector for each eigenvalue.

   Solution: The eigenvalues of A are 6 and -1.
   One corresponding eigenvector for $\lambda = 6$ is $\begin{bmatrix} 4/3 \\ 1 \end{bmatrix}$. Scalar multiples of this vector except for the vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ would be acceptable eigenvectors.
   A corresponding eigenvector in the eigenspace of A for $\lambda = -1$ is $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.
   Again, scalar multiples of this vector except for the zero vector are acceptable eigenvectors.