MATH 54 SAMPLE QUIZ 4 20 SEPTEMBER 2015

You have 20 minutes to take this quiz, for a total of 10 points.

Name:____

1. (*5 points total*) Given $T(x_1, x_2, x_3) = (x_1 - 2x_2 + x_3, -4x_1 + 5x_2 + 6x_3)$

a). (*2 points*) Write down the matrix of T (i.e., write down the matrix A such that $T(\mathbf{x}) = A\mathbf{x}$, for any $\mathbf{x}$).

Solution. $A = \begin{bmatrix} T(\mathbf{e}_1) & T(\mathbf{e}_2) & T(\mathbf{e}_3) \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ -4 & 5 & 6 \end{bmatrix}$ □

b). (*3 points*) Is $T : \mathbb{R}^3 \to \mathbb{R}^2$, as given above, one-to-one? Is it onto? (answer with brief explanation)

Solution. The matrix can have at most 2 pivots and so there cannot be a pivot in every column. Thus, the transformation T can <u>never be one-to-one</u>. Since the matrix A above, has a pivot in each row, the transformation is onto. □

2. (*5 points total*) $A = \begin{bmatrix} 4 & 1 & 3 \\ 2 & 3 & -6 \\ -1 & 1 & -2 \end{bmatrix}$

a). (*3 points*) Calculate the inverse of A by row reduction method.

Solution. $A = \begin{bmatrix} 4 & 1 & 3 & 1 & 0 & 0 \\ 2 & 3 & -6 & 0 & 1 & 0 \\ -1 & 1 & -2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[R_2+2R_3]{R_1+4R_3} \begin{bmatrix} 0 & 5 & -5 & 1 & 0 & 4 \\ 0 & 5 & -10 & 0 & 1 & 2 \\ -1 & 1 & -2 & 0 & 0 & 1 \end{bmatrix}$

$\xrightarrow[R_1 \leftrightarrow -R_3]{R_1-R_2} \begin{bmatrix} 1 & -1 & 2 & 0 & 0 & -1 \\ 0 & 5 & -10 & 0 & 1 & 2 \\ 0 & 0 & 5 & 1 & -1 & 2 \end{bmatrix} \xrightarrow[R_3 \div 5]{R_2+2R_3} \begin{bmatrix} 1 & -1 & 2 & 0 & 0 & -1 \\ 0 & 5 & 0 & 2 & -1 & 6 \\ 0 & 0 & 1 & 1/5 & -1/5 & 2/5 \end{bmatrix}$

$$\xrightarrow[R_2 \div 5]{R_1+R_2/5} \begin{bmatrix} 1 & 0 & 2 & 2/5 & -1/5 & 1/5 \\ 0 & 1 & 0 & 2/5 & -1/5 & 6/5 \\ 0 & 0 & 1 & 1/5 & -1/5 & 2/5 \end{bmatrix} \xrightarrow{R_1-2R_3} \begin{bmatrix} 1 & 0 & 0 & 0 & 1/5 & -3/5 \\ 0 & 1 & 0 & 2/5 & -1/5 & 6/5 \\ 0 & 0 & 1 & 1/5 & -1/5 & 2/5 \end{bmatrix}.$$

Thus $A^{-1} = \frac{1}{5} \begin{bmatrix} 0 & 1 & -3 \\ 2 & -1 & 6 \\ 1 & -1 & 2 \end{bmatrix}$

□

b). (*2 points*) Use this to solve $A\mathbf{x} = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$.

Solution. Since A is invertible, $A\mathbf{x} = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} \Rightarrow \mathbf{x} = A^{-1} \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$

$$= \frac{1}{5} \begin{bmatrix} 0 & 1 & -3 \\ 2 & -1 & 6 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3+12 \\ 2-3-24 \\ 1-3-8 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 15 \\ -25 \\ -10 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ -2 \end{bmatrix}$$

□

The End.