Name: ____________________

Section: ____________________

1. Which of the following sets of vectors in $\mathbb{R}^3$ contain two linearly independent vectors but no more? (Note that, geometrically, this is the same as spanning a plane).

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ -6 \\ 0 \end{pmatrix} \right\}, \quad \left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}, \quad \left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right\}.$$

Solution: The two vectors in the first set are linearly dependent on each other so cannot span a plane. The two vectors in the second set are linearly independent (seen from considering the second entry, which is zero in one and nonzero in the other). The third set contains at least two linearly independent vectors as it contains the second set. However the final vector can be expressed as the difference of the other two and hence is dependent on them.

2. Does $\begin{pmatrix} 5 \\ 1 \\ 5 \end{pmatrix}$ lie in the span of $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$? Deduce whether or not $\begin{cases} 2x_1 + x_2 = 5 \\ x_1 - x_2 = 1 \\ 3x_1 - x_2 = 5 \end{cases}$ has a solution.

Solution: $\begin{pmatrix} 5 \\ 1 \\ 5 \end{pmatrix} = 2 \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + 1 \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ and so it does lie in the stated span. Hence, $x_1 = 2, x_2 = 1$ solves the system.