Name: _____

Section: _____

1. Find the general solution of the linear system corresponding to the following **augmented** matrix.

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

Solution: We apply elementary row operations to the augmented matrix. Firstly, switch two rows, and then multiply -2 to the first row and add it to the second to get

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & -3 & 0 & -3 \end{bmatrix}$$

Then divide the second row by -3 to get the RREF:

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

In terms of equations, we get

$$x_1 + 3x_3 = 2$$
$$x_2 = 1$$

So $x_2 = 1$ and either x_1 or x_3 is free. By taking x_3 as a free variable, the general solution is

$$x_1 = -x_3 + 2$$
$$x_2 = 1$$
$$x_3 = \text{free}$$

2. For what numbers a, b, c is the following matrix in row echelon form (REF) or reduced row echelon form (RREF)?

$$\begin{bmatrix} 0 & a & 1 & b & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & c \end{bmatrix}$$

REF: any a, b, c.

RREF: $a = 1$ and $b = c = 0$; or $a = b = c = 0$.