1. For which values of a is the following matrix invertible?

$$\begin{pmatrix} a & 0 & 1 \\ -1 & a & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

2. Label the following statements as either true or false.
   (a) $\det A^T = \det A$
   (b) A matrix A is invertible if there is another matrix B such that $AB = I$.
   (c) The dimension of a subspace of $\mathbb{R}^n$ is at most n.
   (d) If A and B are invertible $n \times n$ matrices, then $(AB)^{-1} = A^{-1}B^{-1}$.
   (e) If A is a square matrix, then after adding 2 times the first row of A to the second row, the determinant is multiplied by 2.
   (f) Every subspace of $\mathbb{R}^n$ contains at most n vectors.
   (g) If a 3×5 matrix A represents a surjective linear transformation, then $\mathrm{Null}(A)$ must be exactly 2-dimensional.
   (h) If A and B are $n \times n$ matrices and AB is invertible, then BA must be invertible too.

3. A linear transformation, $T : \mathbb{R}^3 \to \mathbb{R}^3$, has the following effect:

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}1\\-1\\2\end{bmatrix}, T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\-1\\1\end{bmatrix}, T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\\0\end{bmatrix}.$$

   (a) What is the standard matrix of the transformation?

   (b) Is the transformation one-to-one? Is it onto?

   (c) Find a basis for the column and null spaces.

4. (a) Let A be a $n \times n$ matrix. Relate $\det(-A)$ to $\det(A)$.

   (b) Suppose A, B are $n \times n$ matrices. If AB is invertible show A and B must both be invertible.

   (c) Suppose $A^k = 0$. Show that A cannot be invertible.

5. Let $\mathcal{B} = \{1, t-1, (t-1)^2\}$ be a subset of $\mathbb{P}_2$.
   (a) Show that $\mathcal{B}$ is a basis for $\mathbb{P}^2$.
   (b) Find the $\mathcal{B}$-coordinate of $1 + 2t + 3t^2$.

6. Let S be the tetrahedron in $\mathbb{R}^3$ with vertices at $(1,1,1),(2,3,4),(3,4,5)$, and $(4,5,7)$. Find its volume.

7. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by rotating points $\frac{\pi}{4}$ radians counterclockwise around the origin, then reflecting them across the y axis. What is the standard matrix of T?

8. $\mathbb{R}[x]$ is the set of polynomials with real coefficients. It is a (real) vector space with the usual addition and scalar multiplication you know and love from high school. Differentiation, $\frac{\mathrm{d}}{\mathrm{d}x} : \mathbb{R}[x] \to \mathbb{R}[x]$ is a linear operator.

   (a) What is $\operatorname{Ker}\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)$?

   (b) What is $\operatorname{Im}\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)$?

   (c) Is $\frac{\mathrm{d}}{\mathrm{d}x}$ injective? Is $\frac{\mathrm{d}}{\mathrm{d}x}$ surjective?

   (d) Is $\dim(\mathbb{R}[x])$ finite? If so, what is it? If not, prove that it is not.