Welcome to Math 54 !

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This week: Friday "Practice Quiz"

Next week: Office Hours Tuesday
1-3pm, location TBA

Why take Math 54?

It's fun!

It's easy!   Linear!

It's powerful! Solve eqns by algorithms – 1) Linear eqns

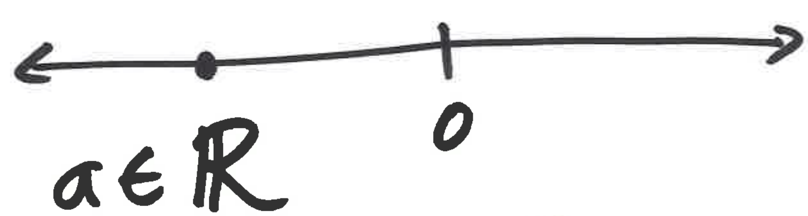
2) Eigenvector / Eigenvalues eqns

3) Diff. eqns

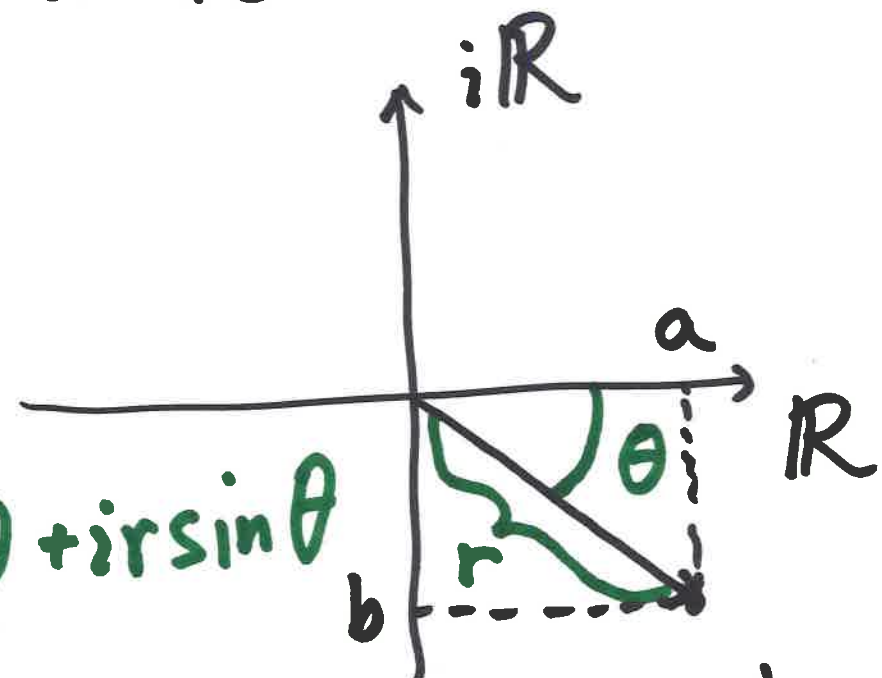
It's sexy! For example, Google's search algorithm is at heart an eigenvalue/eigenvector eqn.

Notation

$\mathbb{R}$ = real numbers



$\mathbb{C}$ = complex numbers



$z = a + ib$
$\in \mathbb{C}$

Polar form: $z = r\cos\theta + ir\sin\theta$
$= re^{i\theta}$

More notation: $a, b, c \ldots$ numbers

$x, y, z, \ldots$ variables

<u>Def</u> A <u>linear eq<u>n</u></u> in n vars is an eq$^{\underline{n}}$ that can be put in form:

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

vars

numbers

Which of the following is a lin. eq.?

1) $3(x_1 - 2x_2) = 5(4 - x_3)$ Yes!

2) $3(x_1^2 - 2x_2) = 7$ No!

3) $(x_1 - 3)^2 = x_1^2 + 5x_2$ Yes!

Key idea of lin. eqn. sums and scales of solns. are again solns. if the number $b = 0$.

__Def__ A __system of lin eqns__ / __linear syst.__ in n vars is a finite collection of lin eqns.

Can be put in form:

$$\left.\begin{array}{ccccc} a_{11}x_1 & + \cdots & + a_{1n}x_n & = & b_1 \\ \vdots & & \vdots & & \vdots \\ a_{m1}x_1 & + \cdots & + a_{mn}x_n & = & b_m \end{array}\right\} m \text{ eqns}$$

Examples of lin systs:

1) $m=2$, $n=3$

$$\begin{aligned} 2x_1 - 3x_2 + 7x_3 &= 5 \\ -x_1 + 0x_2 - 4x_3 &= 0 \end{aligned}$$

2) $m=3$, $n=1$

$$\begin{aligned} -x_1 &= 7 \\ 3x_1 &= 0 \\ 15x_1 &= 2 \end{aligned}$$

<u>Note</u>: $n > m$ $\rightsquigarrow$ easier to solve

$m > n$ $\rightsquigarrow$ harder to solve

__Def__ __Solution set__ of a lin. syst. is the set of all $(s_1, \dots, s_n)$ that solve all the eqns.

"n-tuple" of numbers

__Exer__: Find soln set for

1) $$\begin{aligned} 3x_1 - x_2 &= 0 \\ 2x_1 &= 6 \end{aligned}$$

Soln set = $\{(3, 9)\}$

There is unique soln.

2) $3x_1 + x_2 = 1$

$-6x_1 - 2x_2 = 0$

Soln set = $\emptyset$

No soln.

3) $x_1 - x_2 + x_3 = 0$

$x_2 + x_3 = 0$

Soln set =

$\{(-2s, -s, s)\}$

s any number

Lots of solns.

Three possible outcomes:

1) No solns. – inconsistent

$$\left.\begin{array}{l}\text{2) Unique soln.}\\ \text{3) Infinitely many solns}\end{array}\right\} \text{– consistent}$$

Exer For what number c does the following lin syst satisfy each of the above?

$$x_1 + c x_2 = 1$$
$$2x_1 + 2x_2 = 0$$

Soln to exer. 2nd eqn $\Longrightarrow$ (implies) any soln must be of form $(s, -s)$, s number

Then 1st eqn

$$\Longrightarrow \quad s + c(-s) = 1$$

Solve for s:

$$s(1-c) = 1$$

$$s = \frac{1}{1-c}$$

Conclusion: $c \neq 1$, then there is a unique soln $\left\{ \left(\frac{1}{1-c}, \frac{-1}{1-c} \right) \right\}$

If $c = 1$, then there is no soln.

__Def__. Two lin systs. are __equivalent__ if they have the same soln sets.

__Exer__. For what c are the following lin systs. equivalent?

(I)
$$\begin{aligned} x_1 - cx_2 &= 0 \\ x_1 + x_3 &= 0 \end{aligned}$$

(II)
$$\begin{aligned} 2x_1 - x_2 + x_3 &= 0 \\ x_2 + x_3 &= 0 \end{aligned}$$

Soln set for (I) $= \{(cs, s, -cs)\}$, s number

Soln set for (II) $= \{(-t, -t, t)\}$, t number

Conclusion: sets are same

$\iff$ $\underline{c = 1}$.

(if and only if)

$$\{(s, s, -s)\} = \{(-t, -t, t)\}$$

s number $\qquad$ t number

Take $t = -s$.

Matrix Notation:

$$2x_1 - 3x_2 + 0x_3 - x_4 = 7$$
$$x_1 + x_2 - 3x_3 + 2x_4 = 0$$

$$\rightsquigarrow$$

$$\begin{array}{c} m \\ = \\ 2\text{ rows} \end{array} \left[\begin{array}{cccc:c} 2 & -3 & 0 & -1 & 7 \\ 1 & 1 & -3 & 2 & 0 \end{array}\right]$$

$n+1=5$ cols

<u>Called</u>: augmented matrix of lin syst.