Name (Last, First): ____________________

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1) Find the dimension of the subspace H inside of $\mathbb{R}^4$ given by all vectors of the form

$$\begin{bmatrix} 2a+4b+c+5d \\ a-7b-4c+7d \\ -a+b+c-4d \\ -a-b-3d \end{bmatrix}$$

where a, b, c, d are any real numbers.

Solution: $\begin{bmatrix} 2a+4b+c+5d \\ a-7b-4c+7d \\ -a+b+c-4d \\ -a-b-3d \end{bmatrix} = a \cdot \begin{bmatrix} 2 \\ 1 \\ -1 \\ -1 \end{bmatrix} + b \cdot \begin{bmatrix} 4 \\ -7 \\ 1 \\ -1 \end{bmatrix} + c \cdot \begin{bmatrix} 1 \\ -4 \\ 1 \\ 0 \end{bmatrix} + d \cdot \begin{bmatrix} 5 \\ 7 \\ -4 \\ -3 \end{bmatrix}.$

So $H = Span\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ -4 \\ -3 \end{bmatrix} \}.$

Thus $\dim(H)$ equals the rank of the matrix $A = \begin{bmatrix} 2 & 4 & 1 & 5 \\ 1 & -7 & -4 & 7 \\ -1 & 1 & 1 & -4 \\ -1 & -1 & 0 & -3 \end{bmatrix}.$

A row-reduces to $\begin{bmatrix} 1 & 0 & -0.5 & 3.5 \\ 0 & 1 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, so $\dim(H) = \text{rank}(A) = 2$.

2) If A is a 9×6 matrix, what is the largest possible dimension of the row space of A? What is the largest possible dimension of the null space $Nul(A)$?

Solution: The number of pivots in A cannot be more than the number of rows or the number of columns of A, so A has at most 6 pivots. Since $\dim(Row(A)) = \text{rank}(A) = \dim(Col(A))$, the largest possible value for $\dim(Row(A))$ is also 6.

By the Rank Theorem, $\text{rank}(A)+\dim(Nul(A)) = 6$, so the largest possible value for $\dim(Nul(A))$ is 6, which occurs when $\text{rank}(A) = 0$.