Math 54 **Name (Print):** ______________________

Practice Quiz #4

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1. Suppose $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is the standard matrix for an invertible linear function $T : \mathbb{R}^2 \to \mathbb{R}^2$ and $B = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 4 & 0 \end{bmatrix}$ is the standard matrix for a linear function $S : \mathbb{R}^3 \to \mathbb{R}^2$. Find the standard matrix for $T \circ T \circ T^{-1} \circ S$.

   Solution: First ntoice that T and T^{-1} cancel. That is, $T \circ T \circ T^{-1} \circ S = T \circ I \circ S = T \circ S$. Since the standard matrix for a composition of two linear functions is given by the product of their standard matrices, the matrix for $T \circ S$ is $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 3 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 8 & 4 \\ 15 & 16 & 12 \end{bmatrix}$

2. Let $A = \begin{bmatrix} 1 & -2 & 4 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Find a basis for $Col(A)$ and a basis for $Nul(A)$.

   Solution: A is already in echelon form, so we don't need to do too much work. The first two columns of A are the pivot columns of A, so they form a basis for $Col(A)$.

   To find a basis for $Nul(A)$, we want to parameterize the solution space of $A\mathbf{x} = \mathbf{0}$. This equation corresponds to the system of equations $x_1 - 2x_2 + 4x_3 + x_4 = 0$, $x_2 + x_3 = 0$, and $0 = 0$. So the general solution is given by $x_2 = -x_3$ and $x_1 = -6x_3 - x_4$ with x_3 and x_4 free. This gives us $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -6x_3 - x_4 \\ -x_3 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -6 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$. So $\begin{bmatrix} -6 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ form a basis for $Nul(A)$.