Name (Last, First): ____________________

Student ID: ____________________

1. Find the general solution of the linear system corresponding to the following **augmented** matrix.

$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix}$$

**Solution.** First, convert the matrix into REF by applying the row operations.

$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 & 4 \\ 0 & 0 & -7 & 14 \end{bmatrix} \quad \text{(Replacement)}$$

where the corresponding system of linear equations is

$$\begin{aligned} x_1 - 2x_2 - x_3 &= 4 \\ -7x_3 &= 14 \end{aligned}$$

Therefore, $x_3 = -2$ and $x_1 - 2x_2 = 2$.
If we pick $x_2$ as a free variable, the general solution reads

$$\begin{aligned} x_1 &= 2x_2 + 2 \\ x_2 &= \text{free variable} \\ x_3 &= -2 \end{aligned}$$

2. Determine if **b** is a linear combination of the vectors formed from the columns of the matrix $A$.

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

**Solution.** Note that **b** is a linear combination of the column vectors of $A$ if and only if $A\mathbf{x} = \mathbf{b}$ has a solution.

Calculate a REF of the augmented matrix $[A|\mathbf{b}]$ :

$$[A|\mathbf{b}] = \left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{array}\right]$$
$$\sim \left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

The last row $(0\ 0\ 0 \mid 3)$ indicates that the solution does not exist. Hence, **b** is NOT a linear combination of the column vectors of $A$.