Name (Last, First): ____________________

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1. Find a particular solution to the following differential equation.

$$y'' - 2y' - 3y = 3te^t$$

Solution. We first solve the correpsonding homogeneous differential equation:

$$y'' - 2y' - 3y = 0$$

The auxiliary equation is

$$r^2 - 2r - 3 = 0$$

which has two distinct zeroes 3 and -1. Since 1 is not a zero of the auxiliary equation, $y_p = (at+b)e^t$ will work for some real numbers a and b. We plug in y_p to the differential equation:

$$\begin{aligned} 3te^t &= y_p'' - 2y_p' - 3y_p \\ &= (-4at - 4b)e^t \end{aligned}$$

This implies that $a = -\frac{3}{4}$ and $b = 0$. Therefore,

$$y_p = -\frac{3}{4}te^t$$

is a particular solution to the given differentail equation. □

2. Find a general solution to the following differential equation.

$$y'' + 3y' + 2y = t + e^t$$

Solution. We first solve the correpsonding homogeneous differential equation:

$$y'' + 3y' + 2y = 0$$

The auxiliary equation is

$$r^2 + 3r + 2 = 0$$

which has two distinct zeroes -2 and -1. We then get a general solution y_h of the homogeneous equation:

$$y_h = c_1 e^{-2t} + c_2 e^{-t}$$

where c_1 and c_2 are arbitray real numbers. To find a particular solution for the given differential equation, we break it up into two parts; we find a particular solution for

$$y'' + 3y' + 2y = t$$

and

$$y'' + 3y' + 2y = e^t$$

respectively. And then we add them to obtain a particular solution for the given differential equation. For the former, 0 is not a zero of the auxiliary equation and so $y_{p_1} = at + b$ will work for some real numbers a and b. Plug it in to the former:

$$\begin{aligned} t &= y''_{p_1} + 3y'_{p_1} + 2y_{p_1} \\ &= 2at + 3a + 2b \end{aligned}$$

This implies that $a = \frac{1}{2}$ and $b = -\frac{3}{4}$. So,

$$y_{p_1} = \frac{1}{2}t - \frac{3}{4}$$

is a particular solution to the former.

For the latter, 1 is not a zero of the auxiliary equation and so $y_{p_1} = ce^t$ will work for some real number c. Plug it in to the latter:

$$\begin{aligned} e^t &= y''_{p_2} + 3y'_{p_2} + 2y_{p_2} \\ &= 6ce^t \end{aligned}$$

This implies that $c = \frac{1}{6}$. So,

$$y_{p_2} = \frac{1}{6}e^t$$

is a particular solution to the latter. Therefore,

$$y_p := y_{p_1} + y_{p_2} = \frac{1}{2}t - \frac{3}{4} + \frac{1}{6}e^t$$

is a particular solution to the original differentail equation. All in all,

$$y_h + y_p = c_1 e^{-2t} + c_2 e^{-t} + \frac{1}{2}t - \frac{3}{4} + \frac{1}{6}e^t$$

is a general solution for the differential equation wherer c_1 and c_2 are arbitrary real numbers. $\square$