Name (Last, First): ______________________________

Student ID: ______________________________

Circle your section:

201	Shin	8am	71 Evans	212	Lim	1pm	3105 Etcheverry
202	Cho	8am	75 Evans	213	Tanzer	2pm	35 Evans
203	Shin	9am	105 Latimer	214	Moody	2pm	81 Evans
204	Cho	9am	254 Sutardja Dai	215	Tanzer	3pm	206 Wheeler
205	Zhou	10am	254 Sutardja Dai	216	Moody	3pm	61 Evans
206	Theerakarn	10am	179 Stanley	217	Lim	8am	310 Hearst
207	Theerakarn	11am	179 Stanley	218	Moody	5pm	71 Evans
208	Zhou	11am	254 Sutardja Dai	219	Lee	5pm	3111 Etcheverry
209	Wong	12pm	3 Evans	220	Williams	12pm	289 Cory
210	Tabrizian	12pm	9 Evans	221	Williams	3pm	140 Barrows
211	Wong	1pm	254 Sutardja Dai	222	Williams	2pm	220 Wheeler

If none of the above, please explain: ______________________________

This is a closed book exam, no notes allowed. It consists of 8 problems, each worth 10 points. We will grade all 8 problems, and count your top 6 scores.

Problem	Maximum Score	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total Possible	60	

Problem 1) True or False. Decide if each of the following statements is TRUE or FALSE. You do not need to justify your answers. Write the full word **TRUE** or **FALSE** in the answer box of the chart. (Each correct answer receives 2 points, incorrect answers or blank answers receive 0 points.)

Statement	1	2	3	4	5
Answer					

1) If $y(t)$ satisfies $y'(t) = 5y(t)$, then $y(0) = 1$.

2) If $y(t)$ satisfies $y'(t) = 5y(t)$, then $y(t)$ satisfies $y''(t) - 4y'(t) - 5y(t) = 0$.

3) If an $n \times n$ constant matrix A is not diagonalizable, then the solution set of $\mathbf{y}'(t) = A\mathbf{y}(t)$ has dimension less than n.

4) The initial value problem $ty''(t) + e^t y'(t) + \sin(t)y(t) = \cos(t), y(1) = 1, y'(1) = -1$ has a unique solution on the domain $(0, \infty)$.

5) Two functions $f_1, f_2 : \mathbb{R} \to \mathbb{R}$ are linearly dependent if the Wronskian

$$W[f_1, f_2](t) = \det \begin{bmatrix} f_1(t) & f_2(t) \\ f_1'(t) & f_2'(t) \end{bmatrix}$$

is zero at some point of $\mathbb{R}$.

Problem 2) Multiple Choice. Determine the correct answer to each of the following questions. You do not need to justify your answers. Write the appropriate letter in the answer box of the chart. (Each correct answer receives 2 points, incorrect answers or blank answers receive 0 points.)

Question	1	2	3	4	5
Answer					

1) What is the reduced row echelon form of the matrix $\begin{bmatrix} 0 & 1 & 2 & -1 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 \end{bmatrix}$?

$A)\ \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ $B)\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ $C)\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$D)\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ $E)$ none of the preceding.

2) What is the determinant of the matrix $\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 5 & 0 & 0 & 4 \end{bmatrix}$?

$A)\ -120$ $B)\ -24$ $C)\ 0$ $D)\ 24$ $E)\ 120$

3) What is the first row of the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$?

$A)\ \begin{bmatrix} 0 & -2 & 1 \end{bmatrix}$ $B)\ \begin{bmatrix} 0 & -3 & -6 \end{bmatrix}$ $C)\ \begin{bmatrix} 0 & 6 & -3 \end{bmatrix}$ $D)\ \begin{bmatrix} 2 & 1 & 0 \end{bmatrix}$ $E)$ the inverse does not exist.

4) For what pair of real numbers (c_1, c_2) is the matrix $\begin{bmatrix} 2 & 0 \\ c_1 & c_2 \end{bmatrix}$ diagonalizable?

$A)\ (1,2)$ $B)\ (2,-2)$ $C)\ (-2,2)$ $D)\ (-1,2)$ $E)$ none of the preceding.

5) For what triple of real numbers (a_1, a_2, a_3), does the function

$$\langle \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \rangle = a_1 x_1 y_3 + a_2 x_2 y_2 + a_3 x_3 y_1$$

define an inner product on $\mathbb{R}^3$?

$A)\ (1,0,-1)$ $B)\ (1,-1,1)$ $C)\ (-1,2,1)$ $D)\ (1,2,1)$ $E)$ none of the preceding.

Problem 3) Let V be the vector space of differentiable real-valued functions on the interval $[-1, 1]$ with inner product

$$\langle f, g \rangle = \int_{-1}^{1} f(t)g(t)dt$$

Let W be the subspace of V spanned by $1, t^2$.

1) (5 points) Find an orthogonal basis of W.

2) (5 points) Find the the function in the line spanned by t closest to e^t.

Problem 4) 1) (5 points) Find the general solution of the third order ODE

$$y''' - 2y'' + 2y' = 0$$

2) (5 points) Find the general solution of the third order ODE

$$y''' - 2y'' + 2y' = t$$

Problem 5) Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

1) (5 points) Find the general solution of the equation $\mathbf{y}'(t) = A\mathbf{y}(t)$.

2) (5 points) Find a solution of the equation $\mathbf{y}'(t) = A\mathbf{y}(t)$ such that $\mathbf{y}(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Problem 6) Consider the heat equation on the rod $[0, \pi]$ with temperature zero boundary conditions:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad u(0,t) = u(\pi,t) = 0.$$

1) (5 points) Find the initial temperature if the temperature at time $t = 1$ is given by $-e^{-1}\sin(x)$.

2) (5 points) Prove that if the temperature at $t = 2014$ is zero throughout the rod, then the initial temperature was zero throughout the rod.

Problem 7) (10 points) Consider the cosine Fourier series

$$|\sin(x)| = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$

for any real number x.

Find a_0, a_1, a_2, a_3. (It may be useful to recall $\sin(u)\cos(v) = \frac{1}{2}(\sin(u+v) + \sin(u-v))$.)

Problem 8) (10 points) Decide if the following assertion is TRUE or FALSE. If true, provide a proof; if false provide a counterexample.

Assertion: If $\mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$ and $\mathbf{v}(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$ solve the equation

$$\mathbf{y}'(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathbf{y}(t)$$

with $\mathbf{u}(0) \perp \mathbf{v}(0)$ and $\mathbf{u}(1) \perp \mathbf{v}(1)$, then $\mathbf{u}(t) \perp \mathbf{v}(t)$ for all t.