Name (Last, First): ________________________________

Student ID: ________________________________

Circle your GSI and section:

Scerbo	8am	200 Wheeler
Scerbo	9am	3109 Etcheverry
McIvor	12pm	3107 Etcheverry
McIvor	11am	3102 Etcheverry
Mannisto	12pm	3 Evans
Wayman	1pm	179 Stanley
Wayman	2pm	81 Evans
Forman	2pm	3109 Etcheverry
Forman	4pm	3105 Etcheverry
Melvin	5pm	24 Wheeler
Melvin	4pm	151 Barrows
Mannisto	11am	3113 Etcheverry
McIvor	2pm	179 Stanley

If none of the above, please explain: ________________________________

This is a closed book exam, no notes allowed. It consists of 6 problems, each worth 10 points, of which you must complete 5. **Choose one problem not to be graded by crossing it out in the box below.** You must justify every one of your answers unless otherwise directed.

Problem	Maximum Score	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total Possible	50	

1. Let $P_{\leq 2}(\mathbb{R})$ denote the real vector space of polynomials of degree less than or equal to two. Consider the linear transformation $T : P_{\leq 2}(\mathbb{R}) \to \mathbb{R}^2$ given by

$$T(p(x)) = \begin{pmatrix} p(-1) \\ p(1) \end{pmatrix}$$

a. What is the matrix of T with respect to the basis $1, x, x^2$ of $P_{\leq 2}(\mathbb{R})$ and the standard basis of $\mathbb{R}^2$?

b. Find the dimension of the subspace $U \subset P_{\leq 2}(\mathbb{R})$ of polynomials with $p(-1) = p(1) = 0$. Be sure to justify your answer.

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2. Let $v_1, \ldots, v_n$ be a linearly independent list of vectors of V, and let u_1, u_2 be another linearly independent list of vectors of V. Suppose that u_1 and u_2 are each *not* in $span(v_1, \ldots, v_n)$.

Decide if the following assertion is always true or sometimes false. If always true, provide a proof; if sometimes false, provide a counterexample and justify why it is a counterexample.

Assertion: the list $v_1, \ldots, v_n, u_1, u_2$ is linearly independent.

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3. Let V be a vector space and $U \subset V$ a subspace with $\dim V = n$ and $\dim U = k$.

Let $L \subset L(V, V)$ be the subset of linear transformations $T : V \to V$ such that U is T-invariant.

a. Check that L is a subspace.

b. Calculate $\dim L$.

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4. Let V be a finite-dimensional nonzero complex vector space. For each of the following, decide if it is possible for a linear transformation $T : V \to V$ to satisfy the stated requirements. If yes, give an example; if no, justify why not.

a. T is injective but not surjective.

b. $null(T) = range(T)$.

c. For any basis of V, the corresponding matrix of T is diagonal.

d. There exists a linear transformation $S : V \to V$ such that $ST = \mathrm{Id}_V$ and $TS = 0$.

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5. Let V be a two-dimensional complex vector space, and $T : V \to V$ a linear transformation satisfying $T^4 = -T^2$.

a. What are the three possible eigenvalues of T?

b. Is it possible for one such linear transformation T to have all three possible eigenvalues? Be sure to justify your answer.

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6. Consider a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$. Prove that if $null(T) \cap range(T) = \{0\}$ and $\dim range(T) = 3$, then T has at least 2 distinct eigenvalues.