EXERCISES PERTAINING TO LECTURE 2

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Exercise 2.1. Let k be an algebraically closed field and let X/k be a rational surface. Show that if Y/k is a second surface and $D(X) \simeq D(Y)$ then Y is also rational.

Exercise 2.2. Let X be a smooth genus 1 curve over a field k, let d be an integer, and assume that there exists a universal sheaf $\mathscr{P}_d$ on $X \times \mathrm{Pic}^d_X$. Calculate

$$\Phi^{\mathscr{P}_d}(\mathscr{O}_X) \in D(\mathrm{Pic}^d_X).$$

Exercise 2.3. Let X be a smooth projective variety over a field k and let

$$\rho^1 : \Omega^1_X \to \mathscr{H}^{-1}(\mathrm{L}\Delta^*\Delta_*\mathscr{O}_X)$$

be the isomorphism defined in lecture. Explain how to define the induced map

$$\rho^i : \Omega^i_X \to \mathscr{H}^{-i}(\mathrm{L}\Delta^*\Delta_*\mathscr{O}_X)$$

and show that this map is an isomorphism.

Exercise 2.4. Let X and Y be smooth projective varieties over k and let $\Phi : D(X) \to D(Y)$ be an equivalence. Assume that Φ is given by a sheaf P on $X \times Y$ and furthermore that the induced map

$$A^*(X)_{\mathrm{num}} \to A^*(Y)_{\mathrm{num}}$$

preserves the codimension filtration (here we consider Chow groups modulo numerical equivalence and tensored with $\mathbf{Q}$). Show that Φ is given by a line bundle on the graph of birational isomorphism between X and Y.