Analytic description of moduli spaces,

Ex: Hamiltonian Floer theory without bubbling

$$\bar{\mathcal{M}} = \mathcal{M} \cup \mathcal{M}^{\text{broken}}$$

* $\mathcal{M} = \bar\partial_J^{-1}(0)/\mathrm{Aut}$ $\qquad \begin{array}{c} \mathcal{E} \\ \downarrow \uparrow \bar\partial_J \\ \mathcal{B} \end{array}$ space of maps $\qquad$ "global equivariant Fredholm description"

$$\begin{array}{c} \mathcal{E} \\ \downarrow \\ \mathcal{B} \end{array} = \{u : \mathbb{R}\times S^1 \to M \mid u(s,\cdot) \xrightarrow[s\to\pm\infty]{} \text{per. orbit}\} \ \circlearrowleft u \mapsto \partial_s u + J(\partial_t u - X_H(u)) \quad \circlearrowleft \mathrm{Aut} = \mathbb{R}$$

* $\mathcal{M}^{\text{broken}}$ has strata given by fiber products $\mathcal{M} \times_* \mathcal{M}$, $\mathcal{M}\times_*\mathcal{M}\times_*\mathcal{M}, \dots$

$$\mathcal{M} = \bigcup_{\gamma_-,\gamma_+} \mathcal{M}(\gamma_-,\gamma_+)$$

$$\bar{\mathcal{M}} \setminus \mathcal{M} = \bigcup_{\gamma} \mathcal{M}(\cdot,\gamma)\times\mathcal{M}(\gamma,\cdot) \ \cup \bigcup_{\gamma_1,\gamma_2} \mathcal{M}(\cdot,\gamma_1)\times\mathcal{M}(\gamma_1,\gamma_2)\times\mathcal{M}(\gamma_2,\cdot) \ \cup \ \dots$$

* $\bar{\mathcal{M}}$ has compact, metrizable topology
  - $\to$ $\mathcal{C}^\infty_{\text{loc}}/\mathrm{Aut}$ on $\mathcal{M}$ (equivalent to $W^{1,p}_{\text{loc}}/\mathrm{Aut}$ $\forall p>2$, $\mathcal{C}^\infty_{\text{exp decay}}/\mathrm{Aut}$, ...)
  - $\to$ Gromov convergence for $\mathcal{M} \ni \mathcal{V}_i \longrightarrow \mathcal{V}_\infty \in \bar{\mathcal{M}}\setminus\mathcal{M}$

$$\mathcal{V}_\infty = ([v_-],[v_+]) \qquad E(\mathcal{V}_i) = E(v_-) + E(v_+)$$

$$\left\{\begin{array}{ll} \exists\, v_i \in \mathcal{V}_i : & v_i \to v_- \\ s_i \to \infty & v_i(\cdot + s_i,\cdot) \to v_+ \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} \exists\, u_i \in \mathcal{V}_i \\ R_i \to \infty \end{array} : d(u_i, v_- \#_{R_i} v_+) \to 0\right\}$$

Analytic description of moduli spaces,

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$\overline{\mathcal{M}} = \mathcal{M} \cup \mathcal{M}^{\text{broken}}$

* $\mathcal{M} = \bar{\partial}_J^{-1}(0) / \text{Aut}$ global equivariant Fredholm description
* $\mathcal{M}^{\text{broken}}$ has strata given by fiber products
* $\overline{\mathcal{M}}$ has compact, metrizable topology - near $\mathcal{M}^{\text{broken}} \subset \overline{\mathcal{M}}$ given by pregluing - e.g. $\text{Maps} \underset{ev}{\times} \text{Maps} \times (R_0, \infty) \to \text{Maps}$, $(u_-, u_+, R) \longmapsto u_- \#_R u_+$
* $\mathcal{M}$ has local Fredholm descriptions
* in transverse cases, $\mathcal{M}^{\text{broken}} \subset \overline{\mathcal{M}}$ has (smooth) local charts from gluing - e.g. $g: \mathcal{U} \underset{ev}{\times} \mathcal{V} \times (R_1, \infty] \xrightarrow[\text{local homeo}]{} \overline{\mathcal{M}}$, $\mathcal{U}, \mathcal{V} \subset \mathcal{M}$ precompact

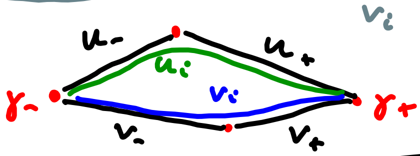
pregluing ↓ almost solutions ↗ Newton iteration ✓

TODO
(1) local injectivity
(2) local surjectivity

(1) <u>injectivity</u>

$$\underbrace{\exp_{u_-\#_{R_i}u_+}(\xi_{R_i})}_{u_i} = \underbrace{\exp_{v_-\#_{S_i}v_+}(\eta_{S_i})}_{v_i}(\cdot + T_i, \cdot)$$

$R_i, S_i \to \infty$



Think of $u: \mathbb{R}\times S^1 \to M$ as $u: \mathbb{R} \to \mathcal{L}M$ and use

$$\boxed{\begin{array}{l}\underline{\text{Gromov-Hausdorff metric}} \text{ on } \{\text{closed subsets of } \underset{\mathcal{L}M}{\underset{\shortparallel}{C^0(S^1,M)}}\} \\ d_{GH}(A,B) = \max\{\sup_{a\in A} d(a,B),\ \sup_{b\in B} d(A,b)\}\end{array}}$$

$$\begin{array}{ccc} \overline{\mathrm{im}\,u_i} & = & \overline{\mathrm{im}\,v_i} \\ ce^{-\delta R_i}\updownarrow & & \updownarrow ce^{-\delta S_i} \\ \gamma_- \in \overline{\mathrm{im}\,u_-} \cup \overline{\mathrm{im}\,u_+} \ni \gamma_+ & & \gamma_- \in \overline{\mathrm{im}\,v_-} \cup \overline{\mathrm{im}\,v_+} \ni \gamma_+ \end{array}$$

& $\left(\begin{array}{c}\underline{\text{unique continuation}} \\ \mathrm{im}\,u \cap \mathrm{im}\,v \neq \emptyset \\ \Rightarrow [u] = [v]\end{array}\right) \Rightarrow \begin{array}{l} u_- = v_- \\ u_+ = v_+ \end{array}$

w.l.o.g.

$$\left.\begin{array}{l} u_-^{-1}(\mathcal{L}M \setminus B_\varepsilon(\text{per. orbits})) = [0, P_-] \\ u_+^{-1}(\quad \text{—— " ——} \quad) = [-P_+, 0] \end{array}\right\} \Rightarrow (u_-\#_R u_+)^{-1}(\text{— " —}) = \underbrace{[-R_i, P_- - R_i] \cup [R_i - P_+, R_i]}_{I(R_i)}$$

$$I(R_i) \longleftarrow u_i^{-1}(\text{— " —}) = v_i^{-1}(\text{— " —}) + T_i \longrightarrow I(S_i)$$

$\Rightarrow T_i, |R_i - S_i| \to 0$

local injectivity $\quad \partial_R g(u_-,u_+,R_1) \neq 0 \in T\mathcal{C}^\infty(\mathbb{R}\times S^1, M) \quad \forall R_1$ suff large

Claim: $\partial_R \zeta_R \xrightarrow[R\to\infty]{} 0$

$f_R(\zeta_R) = 0 \quad \Rightarrow \quad Df_R \cdot \partial_R\zeta_R = -\partial_R f_R(\zeta_R) \longrightarrow 0$

$\zeta_R = Q_R \eta_R \quad \Rightarrow \quad \partial_R \zeta_R = \underbrace{Q_R \dot\eta_R}_{\xi_R} + \underbrace{\dot Q_R}_{\text{bounded}} \eta_R \qquad \eta_R = D_R\zeta_R \longrightarrow 0$

$\eta_R \searrow 0$

$D_R = D_{w_R}\bar\partial_{J,M}$

$$\|\overset{\text{im}\,Q_R}{\xi_R}\| \le C\|D_R\xi_R\| \le \underbrace{C\|D_R - Df_R\|}_{\le 1/2}\|\xi_R\| + \|D f_R(\underbrace{\partial_R\zeta_R}_{\downarrow\, 0} - \underbrace{\dot Q_R\eta_R}_{\downarrow\, 0})\|$$

$\frac{1}{2}\|\xi_R\| \le \ldots\ldots \longrightarrow 0$

$\boxed{\text{Geometric Regularization}}$

(Ex: Hamiltonian Floer without bubbling)

0.) Global Fredholm description of $\widetilde{\mathcal{M}}$

1.) Equivariant Transversality

find J s.t. $\begin{matrix} \mathcal{E} \\ \downarrow \\ \mathcal{B} = C^\infty(\mathbb{R}\times S^1, M) \end{matrix}$ $\bar\partial_{J,H} \pitchfork 0$ $\Rightarrow$ $\bar\partial_{J,H}^{-1}(0) = \bigcup_{\gamma_- \gamma_+} \widetilde{\mathcal{M}}(\gamma_-, \gamma_+) = \widetilde{\mathcal{M}}$

smooth submanifold

2.) Quotient Theorem

$\mathbb{R} \circlearrowright \bar\partial_{J,H}^{-1}(0)$ smooth, proper $\Rightarrow$ $\bar\partial_{J,H}^{-1}(0)/\mathbb{R} = \bigcup_{\gamma_- \gamma_+} \mathcal{M}(\gamma_-, \gamma_+) = \mathcal{M}$

smooth manifold

$\mathcal{M} = \bigcup_{k\geq 0} \mathcal{M}^k$ $\quad \dim \mathcal{M}^k = k$

3.) Gromov Compactness & Gluing

$g : \mathcal{M}^0 \times_{\mathrm{Per}} \mathcal{M}^0 \times (R_1, \infty) \hookrightarrow \mathcal{M}^1$

$\mathcal{M}^1 \setminus \operatorname{im} g$ compact

$\Rightarrow$ $\mathcal{M}^1 \cup_g \mathcal{M}^0 \times_{\mathrm{Per}} \mathcal{M}^0 \times (R_1, \infty] = \overline{\mathcal{M}}^1$

compact 1-mfd, $\partial \overline{\mathcal{M}}^1 \simeq \mathcal{M}^0 \times_{\mathrm{Per}} \mathcal{M}^0$

4.) Cobordism / Continuation map

for $J_0 \neq J_1$ construct $(CF, \partial_0) \simeq (CF, \partial_1)$ from $\mathcal{M}((J_t)_{t\in[0,1]})$

$\leadsto$ steps 1-3 twice more