Math 1A Midterm 2 2004-11-3 2:00-3:30pm.

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 1 mark, which will be given only for a clear correct answer and correct working. There is no partial credit for wrong answers.

1. Differentiate $y = x/\cos(x)$.
2. Differentiate $y = \tan(\cos(x))$.
3. Find dy/dx by implicit differentiation if $x^2y^2 + x\sin(y) = 4$.
4. Find the 43rd derivative of $\sin(2x)$.
5. Use logarithmic differentiation (or any other method you know) to find the derivative of the function $y = x^x$.
6. Find the derivative of $\sinh(x^3)$.
7. Use differentials or a linear approximation to estimate 2.001^3.
8. Find the absolute maximum and absolute minimum values of $f(x) = x^2 - 2x$ on the interval $[0, 3]$.
9. Find all critical numbers of the function $f(x) = x\ln(x)$.
10. Show that the equation $2 + 4x + 2x^3 + 5x^5 = 0$ has exactly one real root.
11. Find the intervals on which f is increasing or decreasing and all local maximum and minimum values of $f(x) = xe^x$.
12. Find the limit $\lim_{x\to 0} x^2/\cos(x)$.
13. Find the limit $\lim_{x\to 0}(\sin(x) - x)/x^3$.

In questions 14 and 15 your sketch should show the domain of the function, local maxima and minima, where the function is increasing or decreasing, any zeros of the function, the behavior for large values of $|x|$, and the behavior near $x = 0$. You need not show convexity or points of inflection.

14. Sketch the curve $y = x - 3x^{1/3}$.
15. Sketch the curve $y = (x - 1)/x^2$.

Solutions: 1. $(\cos(x) + x\sin(x))/\cos(x)^2$. 2. $-\sin(x)/\cos(\cos(x))^2$ 3. $(-2xy^2 - \sin(y))/(2yx^2 + x\cos(y))$. 4. $-2^{43}\cos(2x)$ 5. $x^x(\ln(x) + 1)$. 6. $3x^2\cosh(x^3)$. 7. 8.012. 8. Min is $f(1) = -1$, max is $f(3) = 3$. 9. $x = 1/e$. 10. At least one root by intermediate value theorem, as $f(-1) < 0$, $f(0) > 1$. At most one root by Rolle's theorem, as $f'(x) > 0$ for all x. 11. Decreasing for $x < -1$, increasing for $x > -1$, local min at $x = -1$. 12. 0 13. $-1/6$. 14. See Stewart 4.5.27. 15. See Stewart 4.5.15.