Math 1A Practice Midterm 1.

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for a clear and correct answer in simplified form. There are questions on both sides of the paper. Time allowed: 1.5 hours.

1. Find the domain of the function $f(x) = x/(3x-1)$.

2. Sketch the graph of $y = |\cos(x)|$ for $-8 \leq x \leq 8$.

3. Find a formula for the inverse of the function $f(x) = (4x-1)/(2x+3)$.

4. Sketch the graph of a function f that satisfies the conditions

$$\lim_{x \to 3^+} f(x) = 1, \ \lim_{x \to 3^-} f(x) = 2, \ \lim_{x \to 2^-} f(x) = 0, \ \lim_{x \to 2^+} f(x) = 1, f(2) = 1.$$

5. Evaluate the limit
$$\lim_{x \to 1} \sqrt{x^4 + 3x + 4}$$

6. Find a positive number δ such that $|1/x - \cdot 5| < \cdot 1$ whenever $|x - 2| < \delta$.

7. Find the numbers at which f is discontinuous, where f is defined by $f(x) = x + 1$ if $x \leq 1$, $f(x) = 1/x$ if $1 < x < 3$, $f(x) = \sqrt{x-3}$ if $x \geq 3$.

8. What is
$$\lim_{x \to +\infty} \frac{4x^2 - 3}{5x^2 - 7x + 100}$$

9. A curve has equation $y = f(x)$. Write and expression for the slope of the secant line through the points $(3, f(3))$ and $(x, f(x))$, and write an expression for the slope of the tangent line at $(3, f(3))$.

10. If $g(x) = 2 - x^3$, find $g'(0)$ and use it to find an equation of the tangent line to the curve $y = 2 - x^3$ at the point $(0, 2)$.

11. Find the derivative of the function $f(x) = x^3$ using the definition of derivative.

12. Differentiate the function $y = \sqrt{x} - 3e^x$

13. Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 5$ where the tangent is horizontal.

14. Differentiate $x^7 e^x$

15. Differentiate
$$\frac{\sqrt{x}}{e^x + 1}$$

Solutions:

1. All reals other than 1/3. 2. 3. $x = (3y+1)/(4-2y)$ where $y = f(x)$. 4. 5. $\sqrt{8}$ 6. Any δ less than or equal to 1/3 will do. 7. 1 and 3 8. 4/5 9. $(f(x) - f(3))/(x-3)$, $\lim_{x \to 3}(f(x) - f(3))/(x-3)$ (or $\lim_{h \to 0}(f(3+h) - f(3))/h$). 10. $g'(0) = 0$, tangent line is $y = 2$. 11. $f'(x) = \lim_{h \to 0}(x+h)^3 - x^3 = \lim_{h \to 0} 3x^2 h + 3xh^2 + h^3 = 3x^2$. 12. $x^{-1/2}/2 - 3e^x$ 13. $x = 1$ or -2 14. $7x^6 e^x + x^7 e^x$ 15. $((e^x + 1)x^{-1/2}/2 - x^{1/2}e^x)/(e^x + 1)^2$