Practice problems for Midterm 1, Math 1A, section 1

1. Let $F(x) = \sqrt{2+x}$, $G(x) = \sqrt{2-x}$. Find $F + G$, FG, F/G, and $G \circ F$, and find their domains. Determine which of these functions is even, odd, or neither.

2. Draw the graph of $y = x^2$. Use the graph to find a number δ such that if $|x - 1| < \delta$, then $|x^2 - 1| < .96 = \frac{24}{25}$. Label the corresponding intervals on your graph.

3. Let $f(x) = \frac{x+8}{x^2-4}$

   (a) What is the domain of f?

   (b) Find $f(1)$, $f(-3)$, and the x- and y-intercepts of f.

   (c) Is f even, odd, or neither? Give an explanation.

   (d) Find $\lim\limits_{x \to \infty} f(x)$, $\lim\limits_{x \to 2^+} f(x)$, $\lim\limits_{x \to 2^-} f(x)$. What are the asymptotes of $y = f(x)$?

   (e) Sketch all of the points and asymptotes you have found from the previous parts on a graph. Then sketch the graph of $y = f(x)$ on the same graph.

4. Let
$$h(x) = \begin{cases} x^2 + 2a, & x \le 1 \\ ax + 3, & x > 1 \end{cases} \tag{1}$$

   Determine a so that h is continuous for all real numbers. Explain with upper and lower limits.

5. Prove rigorously the following limit, using the M-N definition of an infinite limit:

$$\lim_{x \to \infty} x - 100 \cos x = \infty$$

6. Find the tangent line to the graph of $y = 2x^3 - 5x$ at the point $(-1, 3)$.

7. Find $g'(x)$, where $g(x) = \sqrt{x-2}$ using the limit definition of the derivative and the methods for finding limits that we have developed so far. What are the domains of $g(x)$ and $g'(x)$?