

Math 1A Practice Final

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for a clear and correct answer in simplified form. There are questions on both sides of the paper.

The questions on this practice final are all exercises in Stewart; the exercise number is given at the end of the question so you can check your answer.

1. Draw the graph of $y = \sqrt{x+3}$. (1.3.17)
2. Prove that $\lim_{x \rightarrow 0} x^2 = 0$ using the ϵ, δ definition of limit. (2.4.25)
3. Prove that $e^x = 2 - x$ has at least one real root. (2.5.51)
4. Differentiate e^x/x^2 . (3.2.5)
5. Find the derivative of $\tan(\cos(x))$. (3.5.29)
6. Find dy/dx if $x^2y + xy^2 = 3x$. (3.6.9)
7. Find the absolute maximum and absolute minimum values of $x/(x^2 + 1)$ on $[0, 2]$. (4.1.53)
8. Prove that $2 \sin^{-1} x = \cos^{-1}(1 - 2x^2)$ for $0 \leq x \leq 1$. (4.2.32)
9. Find $\lim_{x \rightarrow 1^+} \ln(x) \tan(\pi x/2)$. (4.4.43)
10. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle. (4.7.21)
11. Find the point on the line $y = 4x + 7$ that is closest to the origin. (4.7.15)
12. Use Newton's method to find $30^{1/3}$ to two decimal places. (4.9.11)
13. Find the most general anti-derivative of $5x^{1/4} - 7x^{3/4}$. (4.10.5)
14. Find f given that $f''(x) = 2 - 12x$, $f(0) = 9$, $f(2) = 15$. (4.10.37)
15. Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right endpoints. (5.1.5a)
16. Find an expression for the area under the graph of $f(x) = x \cos(x)$, $0 \leq x \leq \pi/2$, as a limit. (5.1.19)
17. Evaluate the integral $\int_{-3}^0 (1 + \sqrt{9 - x^2}) dx$ by interpreting it as an area. (5.2.37)
18. Prove that $\int_0^{\pi/4} \sin^3(x) dx \leq \int_0^{\pi/4} \sin^2(x) dx$. (5.2.51)
19. Find the derivative of $g(x) = \int_0^x \sqrt{1 + 2t} dt$. (5.3.7)
20. Find the derivative of $y = \int_{\sqrt{x}}^{x^3} \sqrt{t} \sin(t) dt$. (5.3.51)
21. Evaluate the integral $\int_0^2 (6x^2 - 4x + 5) dx$. (5.4.17)
22. Evaluate the integral $\int_0^{\pi/4} ((1 + \cos^2(\theta))/\cos^2(\theta)) d\theta$. (5.4.33)
23. Evaluate the indefinite integral $\int \frac{1+4x}{\sqrt{1+x+2x^2}} dx$. (5.5.11)
24. Evaluate the indefinite integral $\int \cot(x) dx$. (5.5.35)
25. Evaluate the definite integral $\int_0^2 (x - 1)^{25} dx$. (5.5.49)
26. Show that $1/2 + 1/3 + \dots + 1/n < \ln(n)$. (5.6.3)
27. Find the area enclosed by the curves $x = 2y^2$, $x + y = 1$. (6.1.17)
28. Find the volume of the region obtained by rotating $y = x^2$, $0 \leq x \leq 2$, $y = 4$, $x = 0$, about the y -axis. (6.2.5)
29. Use the method of cylindrical shells to find the volume of a sphere of radius r . (6.3.43)
30. Find the average value of $(x - 3)^2$ on $[2, 5]$. (6.5.9a)