Sample Final Exam 2

You are allowed one 8.5×11 sheet of notes with writing on both sides. This sheet must be turned in with your exam. *Calculators are not allowed.*

0. (1 point) write your name, section number, and GSI's name on your exam.

1. (3 points) give precise definitions of the following statements or expressions:
(a) $f(x)$ is neither even nor odd
(b) $\int f(x)\,dx$
(c) $\int_a^b f(x)\,dx$

2. (4 points) Show that the tangent lines to the curves $y = x^3$ and $x^2 + 3y^2 = 1$ are perpendicular where the curves intersect.

3. (3 points) Evaluate $\displaystyle\int_0^1 \frac{\tan^{-1} x}{1 + x^2}\,dx$.

4. If f is continuous and $\int_0^4 f(x)\,dx = 6$, find $\int_0^2 f(2x)\,dx$.

5. (5 points) A right circular cone of height h and base radius R has a hole of radius r drilled through its center (from the tip to the center of the base). Find the volume of the solid that remains.

6. (5 points) Let $f(x) = \tanh^{-1}(\sin x)$ and $g(x) = \ln|\sec x + \tan x|$. Compute $f'(x)$, $g'(x)$, $f(n\pi)$ and $g(n\pi)$ with n an integer. What do you conclude?

7. (5 points) A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 10 km/h and reaches the same dock at 3:00 PM. At what time were the two boats closest together? Verify that the distance was minimized using one of the derivative tests.

8. (6 points) Let $f(x) = \dfrac{x^2(\sqrt{x^2+3} - x - 1)}{x^2 - 1}$.
(a) find all vertical and horizontal asymptotes of f.
(b) show that $y = -2x - 1$ is a slant asymptote, i.e. $\lim_{x \to -\infty} [f(x) - (-2x - 1)] = 0$.
Hint for (b): first show that $\lim_{x\to-\infty}[\sqrt{x^2+3} + x] = 0$, *then manipulate* $[f(x) + 2x + 1]$ *to make use of this.*

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9. (9 points) A model rocket is fired vertically upward from rest. Its acceleration (in $\mathrm{m/s^2}$) for the first two seconds is $a(t) = 24t$, at which time the fuel is exhausted and it becomes a freely "falling" body (with constant acceleration $a(t) = -8\,\mathrm{m/s^2}$; the earth's gravity was unusually weak that day.) 10 seconds later, the parachute opens and the velocity v (which is negative at this point) slows according to the differential equation

$$\frac{dv}{dt} = -(v - v_s), \qquad v_s = -5\,\mathrm{m/s^2} \tag{1}$$

until it hits the ground.

(a) Determine the position $s(t)$, velocity $v(t)$, and acceleration $a(t)$ for $0 \leq t \leq 12$. (The parachute opens at $t = 12$).

(b) At what time does the rocket reach its maximum height, and what is that height?

(c) Find $v(t)$ for $t \in [12, T]$, where T is the time when the rocket hits the ground. (you don't have to compute T, which turns out to be very close to 29).

(d) sketch the graphs of $a(t)$, $v(t)$ and $s(t)$ from $0 \leq t \leq T$. Be sure your curves are qualitatively correct even though you did not work out the formulas for $s(t)$ or $a(t)$ for $t > 12$.