

Sample Final Exam 1

You are allowed one 8.5×11 sheet of notes with writing on both sides. This sheet must be turned in with your exam. *Calculators are not allowed.*

0. (1 point) write your name, section number, and GSI's name on your exam.
1. (3 points) give precise definitions of the following statements:
 - (a) $\lim_{x \rightarrow 3^-} f(x) = 17$. (δ - ϵ definition)
 - (b) $f(x)$ is continuous at x_0
 - (c) $f(x)$ has an absolute maximum at x_0 over the interval $[a, b]$
2. (5 points) Evaluate the integral: $\int_0^{\sinh^{-1}(4/3)} e^{\cosh x} \sinh x \, dx$
3. (6 points) Let $f(x) = \begin{cases} \frac{\sin x}{x} & x \neq 0, \\ 1 & x = 0. \end{cases}$
 - (a) Use the definition of the derivative to evaluate $f'(0)$.
 - (b) compute $f'(x)$ for $x \neq 0$.
 - (c) show that $f'(x)$ is continuous at $x = 0$.
4. (5 points) Let $f(x) = \frac{x}{\sqrt{1+x^2}}$. Explain what happens if you try to use Newton's method to solve $f(x) = 0$ with a starting guess $x_0 = 1$.
5. (5 points) Compute the volume of the solid obtained by revolving the region between the parabola $y = (x - 2)^2$ and the line $y = x$ about the y -axis.
6. (3 points) Suppose $g(y)$ is defined to be the value of x such that $x^5 + x + 1 = y$. For example, $g(-1) = -1$, $g(1) = 0$, $g(3) = 1$ and $g(35) = 2$. Evaluate $(g^{-1})'(1)$.
7. (5 points) An airplane flies from New York to Los Angeles, a 3000 mile trip. It costs \$2500 per hour to pay the crew and use the plane. The cost of fuel (in cents per mile) is equal to the velocity of the plane (in miles per hour). For example, it costs \$6 per mile to travel 600 miles/hour. How fast should the plane fly to minimize the cost of the trip?
8. (5 points) Compute the derivative: $\frac{d}{dx} \int_{-x}^{\sqrt{\ln x}} e^{-t^2} \, dt$, ($x > 1$)
9. (3 points) Evaluate the limit by interpreting it as an integral:

$$\lim_{n \rightarrow \infty} \left[\left(1 + \frac{2}{n}\right)^9 \frac{2}{n} + \left(1 + \frac{4}{n}\right)^9 \frac{2}{n} + \left(1 + \frac{6}{n}\right)^9 \frac{2}{n} + \cdots + \left(1 + \frac{2n}{n}\right)^9 \frac{2}{n} \right].$$